THE UNIVERSITY OF ALBERTA

STRATEGIES OF PROBLEM SOLVING AND

RELATED VARIABLES

BY

© MARJORIE LORELLA AFFOLTER

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF EDUCATION

DEPARTMENT OF ELEMENTARY EDUCATION

EDMONTON, ALBERTA

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled " Strategies of Problem Solving and Related Variables" submitted by Marjorie Adella Affolter, in partial fulfilment of the requirements for the degree of Master of Education.

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ABSTRACT

The purpose of this study was to investigate the effectiveness of an inquiry approach to the teaching of arithmetic for the development of efficient strategies of problem solving and to examine relationships among language achievement, tentative thinking and problem solving.

Eighty grade six students, ten from each of eight classrooms located in seven schools, participated in the study. Four of these classes had been identified as using an inquiry approach and four as using a non-inquiry, or traditional, approach. During an oral interview, each subject was administered a Strategies of Problem Solving Test which consisted of six mathematical problems for which an algorithmic solution had not previously been taught. Subjects were asked to think aloud as they attempted to solve the problems, to explain their reasons for suggesting a solution and to give reasons for believing that they had reached a correct or an incorrect solution. Scores were assigned on the basis of the strategies used to solve the problems. The strategies were found to be related to the manner in which the subjects sensed a problem, predicted or hypothesized possible solutions and verified or validated their predictions. Mean scores for the Inquiry and Non-Inquiry groups were compared statistically to determine the significance of any difference observed between them.

Other variables on which the groups were compared were age, intelligence, language achievement and problem solving achievement as measured by the Iowa Problem Solving Subtest. The tape-recorded protocols of the problem solving interviews were examined for evidence of tentativeness, defined as the ability to consider more than one alternative, to suspend judgment and to engage in search behavior.

It was found that subjects in the Inquiry group scored significantly higher on the Strategies Test than did those in the Non-Inquiry group. However, an analysis of the differences in means among the subgroups revealed that statistically significant differences existed only between one Inquiry subgroup and one Non-Inquiry subgroup. This suggested that caution should be used in interpreting the findings. There was no conclusive evidence that students taught by an inquiry approach did in fact develop superior strategies of problem solving as compared to students who had not been taught by this approach.

The most interesting finding as a result of the investigation was that tentativeness, as defined in the study, was closely related to problem solving ability and to the use of trial and error as a technique for solving problems. Students who were able to suspend judgment as to the correctness of a solution until they were able to verify it, were generally the most successful problem solvers. On the other hand, students who considered only one alternative and relied on the correctness of their computation to verify a solution were usually less successful.

Several implications for education were considered and suggestions made for further research. The conclusion was reached that tentative thinking and problem solving may require the learning of a particular function of language and of those language skills which are related to the formulation of hypotheses and to the settling of conjectures.

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THE PROBLEM, ITS NATURE AND SIGNIFICANCE

It is generally recognized that the teacher's goal in the development of problem solving skills is broader than just teaching pupils to solve any given set of problems successfully. Hudgins (1966) believes that the teacher must be concerned with developing flexible strategies so that pupils may learn to attack problems independently, "when cues ordinarily available from the context of the lesson or from the teacher's guidance are missing" (p. 40).

Gagné (1967), recognizing problem solving as the highest form of learning, expresses the belief that when individuals are engaged in problem solving they learn to instruct themselves, to adopt strategies which guide their thinking. Strategies of inquiry play an important role in the theories of both Piaget (1960) and Bruner (1966). Larson (1966) succeeded in identifying several components of the strategies fifth-grade children use as they attempt to solve problems.

The question arises as to what kind of teaching, particularly in the arithmetic classroom, might facilitate the development of useful and flexible strategies of problem solving. Would an 'inquiry' approach to the teaching of mathematics assist this development? Recent authors (Cambridge Conference, 1963; Avital and Shettleworth, 1968) have indicated that it would.

> The discovery approach, in which the student is asked to explore a situation in his own way, is invaluable in developing creative and independent thinking in the individual (Cambridge Conference, 1963, p. 17).

If strategies of problem solving can be developed in the class-

room, it would seem that children who were taught by means of a discovery or 'inquiry' approach to mathematics would learn more efficient strategies than their counterparts in a classroom which utilized a more traditional textbook approach.

Since the process of problem solving is practically synonomous with thinking (Dewey, 1933; Spitzer, 1967), finding a way to investigate the processes (strategies) of problem solving would mean finding some way to monitor the thinking process. Kilpatrick (1967) recommends the "thinking-aloud" technique as a means of studying the protocols which subjects use as they engage in problem solving. In this context it might also be possible to examine the relationship of language and thought in the development of problem solving ability.

Loban (1963) suggested that there might be a relationship between language achievement in school children and the degree of tentative thinking in which they engage. Studies by Bernstein (1967) and Halliday (1969) also suggest that skills which are required for "higher-order" problem solving, such as the formulation of hypotheses, prediction of outcomes and validation, may not be within the grasp of the child who is deficient or restricted in language.

It seemed that the kind of research needed at the present time in the area of problem solving in arithmetic was an investigation into the <u>strategies</u> which children use as they attempt to solve a set of mathematical problems for which they had not previously learned a straightforward or 'pat' solution. Since problem solving processes can only be studied by having subjects generate observable sequences of behavior (Kilpatrick, 1969), the administration of these problems in

an interview situation was indicated; at the same time the relationship of language, tentative thinking and problem solving could also be studied.

I. PURPOSE OF THE STUDY

The purpose of this study was to investigate the problem solving behavior of grade six school children and two seemingly related variables: the kind of arithmetic program which was being followed and the language achievement of the pupils in the study. A secondary purpose was to investigate the relationship of tentative thinking to both language achievement and problem solving. Specifically, the research was intended to answer these questions:

1. Are there identifiable strategies which children adopt as they engage in a search for a problem solution?

2. Does an inquiry approach to mathematics teaching have any significant effect on the strategies used by students in an 'inquiry' classroom?

3. Are high achievers in language more successful in problem solving than low achievers?

4. Is tentativeness related to successful strategies of problem solving or to language achievement?

5. Do high achievers on a conventional arithmetic problem test exhibit more efficient strategies when attempting to solve a "higher-order" problem than do low achievers?

6. Are age, sex and intelligence significant factors in determining problem solving ability?

II. DEFINITION OF TERMS

Verbal Problem of Arithmetic. A quantitative situation described in words in which a definite question is raised but for which the arithmetical operation is not indicated (Spitzer, 1948, p. 209).

Arithmetic Problem Solving Achievement. In this study, arithmetic problem solving achievement will be defined as a subject's score on the Arithmetic Problem Solving Subtest of the Iowa Tests of Basic Skills.

Higher-Order Problem. A problem which requires the combining of two or more previously learned principles or rules to obtain a solution to a problem for which an algorithmic solution has not been taught.

Strategies of Problem Solving. For the purpose of this study, the strategies of problem solving will be defined as the process of orienting oneself to a problem, predicting or suggesting possible solutions and verifying the result.

Problem Solving Ability. In this study, problem solving ability will be defined as the subject's score on the Strategies of Problem Solving Test constructed by the investigator.

Inquiry Classroom. A classroom which does not use a 'textbook' approach to the teaching of arithmetic; where many activities and investigations have been set up to encourage pupils to seek and verify their own answers to problems of a mathematical nature; where students are encouraged to discover mathematical principles and relationships by means of a program of planned activities.

Non-Inquiry Classroom. A classroom in which a conventional 'textbook' approach is used; where students do a series of assigned

exercises which are then corrected by the teacher; where the teacher uses primarily an expository mode of teaching: i. e. teaches a rule or algorithm and then provides practice in applying the rule; where students seldom engage in investigations or mathematical activities of an exploratory nature.

Intelligence. For the purposes of this study, a subject's intelligence will be defined as his I. Q. on the Lorge-Thorndike Tests of Intelligence. It will consist of two separate measures, one verbal and one non-verbal.

Tentativeness. A tendency on the part of an individual to conjecture or to predict, followed by efforts to validate predictions or to settle conjectures; to reject a previous conclusion or prediction when the evidence warrants it; to suspend judgement as the "rightness" of a solution until verification of some kind is made.

Language Achievement. For the purposes of this study, language achievement will be defined by a rating on a written assignment together with a grade assigned by the language teacher based on the pupil's oral and written language competency.

III. THE HYPOTHESES

In order to answer the questions posed with regards to the effect of an 'inquiry' approach on the problem solving strategies of pupils and to investigate the relationship of language achievement, of tentativeness and of other variables related to problem solving ability, the following null hypotheses were made:

Hypothesis 1. There is no significant difference in problem solving ability, as measured by the Strategies of Problem Solving

Test, for students in the Inquiry group and those in the Non-Inquiry group, when controlling for age, intelligence and problem solving achievement.

Hypothesis 2. There is no significant difference in problem solving ability as measured by the Strategies of Problem Solving Test for male and female subjects in the study.

Hypothesis 3. There is no significant relationship between a subject's problem solving ability as measured by the Strategies of Problem Solving Test and:

(a) Language Achievement

(b) Verbal Intelligence

(c) Non-Verbal Intelligence

(d) Age

(e) Arithmetic Problem Solving Achievement.

Hypothesis 4. There is no significant difference between subjects classified as tentative and those classified as non-tentative in:

(a) Language Achievement

(b) Problem Solving Ability.

IV. DESIGN OF THE STUDY

The population from which the sample was drawn consisted of eight grade six classrooms from the Edmonton Public Schools. Four of these classrooms were identified as following an 'inquiry' approach and four as following a 'non-inquiry' approach. The size of each class was approximately the same and about the same amount of time was spent each day on arithmetic instruction.

The sample used in the study consisted of ten subjects randomly chosen from each of the eight classrooms in the study: forty subjects in the Inquiry group and forty in the Non-Inquiry group.

A *Strategies of Problem Solving Test* was constructed by the investigator and administered during a tape-recorded interview with each of the subjects. The tape recordings were transcribed and a score assigned to each subject on the basis of the strategies used to solve the problems. To test the hypotheses the mean score of the Inquiry subjects was compared to the mean score of the Non-Inquiry group and the difference submitted to a statistical test to determine its significance.

Data related to the remaining hypotheses were collected and analyzed statistically. These data included the Arithmetic Problem Solving Subtest of the *Iowa Tests of Basic Skills*; verbal and non-verbal intelligence quotients based on the *Canadian Lorge Thorndike Tests of Intelligence*; a language rating based on a written composition and on the classroom teacher's evaluation; and a subjective analysis of tentativeness during the problem solving sessions.

V. BASIC ASSUMPTIONS

The investigation was based upon the following assumptions:

1. It was assumed that the problems chosen for the *Strategies of Problem Solving Test* would elicit observable problem solving behavior on the part of the subjects.

2. It was assumed that the strategies used by the subjects and observed by the investigator were indicative of the problem solving ability of the subjects.

3. It was further assumed that the investigator could make an accurate assessment of the kind of program which obtained in the eight classrooms in the study.

VI. LIMITATIONS

The study had the following limitations:

1. Many variables which may affect problem solving, such as computational skill, reading comprehension, curiosity, persistence, previous success in problem solving, or attitude towards arithmetic, are not controlled in the study.

2. The results of this study will be applicable only to classrooms which are similar to those used in the investigation.

VII. SIGNIFICANCE OF THE STUDY

1. There is a need for an examination of effects of innovative practices such as some of those described in the Inquiry classrooms, before such innovations are widely adopted. The findings of this study may contribute to such an assessment.

2. The study may have curricular implications for the place of verbal problems in an arithmetic program as well as for the kinds of problems which may help to foster the growth of problem solving skills.

3. Researchers in the area of problem solving (Brian, 1966, and Kilpatrick, 1969) have indicated that the component behaviors of mathematical problem solving can best be measured by oral examination and that there is a need for this kind of investigation. The present study may contribute to this needed research.

4. Little is known of the relationship of language and language

devolopment to the skills and abilities required for successful problem solving. Halliday (1969) has suggested that the ability to use language to explore and to search for a solution to a problem may involve learning a particular use of language. The present study may provide direction for further research in this important area of education.

VIII. ORGANIZATION OF THE REPORT

In Chapter One the problem has been introduced. Chapter Two contains a theoretical discussion of the nature of problem solving, of inquiry and discovery learning and of the relationship of language and thought in the process of problem solving, followed by a summary of the research in the area of problem solving processes and of the factors which contribute to the development of problem solving ability. Chapter Three describes the design of the study, the procedures which were followed in identifying and selecting the sample, the instrumentation and the analyses used to test the hypotheses. Chapter Four is devoted to a detailed discussion of the Strategies of Problem Solving Test, its construction and administration, the rationale for the items chosen and the procedures followed, the criteria used for scoring the test and the method of establishing the reliability of the scoring. Chapter Five contains the data analyses and other findings of the study. Finally, Chapter Six presents a summary of the study, the conclusions and implications drawn from the findings, as well as some suggestions for further research.

CHAPTER TWO

REVIEW OF RELATED LITERATURE AND RESEARCH

While educators have long recognized the importance of developing to the fullest potential the problem solving skills of students, research into the phenomenon of problem solving has, for the most part, yielded conflicting results (Monroe, 1928; Lazerte, 1933; Corle, 1958; Miller, 1960; Brian, 1966; and Kilpatrick, 1967). In particular, such research has failed to shed much light upon the actual processes involved in problem solving or upon the best means of fostering the growth of problem solving ability in the arithmetic classroom (Kilpatrick, 1969).

The purpose of this chapter is to consider literature related to problem solving, to discovery teaching and to the role of language in tentative thinking and in problem solving. The first section of the chapter contains a theoretical discussion of the nature of problem solving and its relationship to the verbal problem of arithmetic. The second part examines the literature on discovery (inquiry) teaching and learning. The third discusses the relationship of language and thought in the development of problem solving skills. The fourth section contains a review of related research. Finally, there is a summary of the factors which seem to affect the development of problem solving ability.

I. PROBLEM SOLVING AND THE VERBAL PROBLEM OF ARITHMETIC

What is Problem Solving?

Half a century of research on problem solving has failed to produce a more acceptable definition of the phenomenon known as problem solving than the interpretation by Dewey (1933) of the act of reflective thinking, which he described as occurring in five phases:

1. There is a problem-presenting situation, or dissatisfaction, which occurs when an individual is in a situation in which his previous knowledge does not give him a satisfactory answer.

2. There is an analysis or examination within the mind of the individual, or intellectualization, of the situation in which there is dissatisfaction.

3. A search is conducted for tentative or guiding hypotheses which will remove the dissatisfaction.

4. The fourth phase is deduction: the elaboration of the hypotheses by reasoning or prediction of outcomes so that one of the hypotheses may be selected.

5. The final phase is that of action on the basis of the particular hypothesis selected in step four, thereby providing for the removal of the dissatisfaction (pp. 106-115)

It is obvious from Dewey's interpretation that problem solving is a highly personal activity for, as Cronbach (1948) pointed out, "it is not the question that makes the problem, but rather the person's accepting it as something he must try to solve (p. 34)."

Most of the definitions of problem solving since 1933 are essentially a re-wording or reorganization of the one given by Dewey. Although Polya (1957) uses only four phases to describe a problem

solving encounter, the similarity to Dewey is unmistakable. According to Polya, an individual tries to understand the problem, devises a plan for connecting the given with the unknown, carries out the plan, and then examines the solution to see if the result is verifiable (p.xvi).

In 1953 Henderson and Pingry listed the conditions for problem solving whereby an individual becomes <u>aware</u> of a problem for which he desires to obtain a solution, identifies various <u>hypotheses</u> and then <u>tests</u> these for feasibility.

Kilpatrick (1967) expressed the belief that all learning is essentially problem solving. Gagné (1964, 1967), on the other hand, sees many simpler types of learning which must take place prior to the occurrence of problem solving which is the highest form of learning; i.e. "a set of events which must have been preceded by learning (1964, p. 293)."

Gagné's idea that problem solving operates on previous learnings is consistent with Duncker's theory of a "search model" which bridges the gap between what is given and what is required and serves to direct the individual's thinking.

> His region of search consists of the mathematical concepts and generalizations he has learned. These are appraised and selected in terms of their usefulness . . . If they provide any hunches or suggestions for plans of procedure, the plans are tested to see whether they provide a solution to the problem (in Henderson and Pingry, 1953, p. 240).

Duncker also noted that the production and retention of thought material, an integral part of all problem solving, seemed to be dependent upon the individual's immediate memory span. Some individuals, he observed, failed to make use of their previous learnings and experienced difficulty in remembering the unsuccessful things they

had done in trying to solve the problem (p. 241).

For Bruner (1966) problem solving depends largely upon how an individual processes (acquires, retains and utilizes) the information which is available to him. Bruner concludes that, since problem solving is dependent upon the exploration of alternatives, instruction should facilitate and regulate this exploratory activity on the part of the learner (p. 43).

Regardless of whose definition of problem solving one chooses to consider, it is agreed that problem solving is both a legitimate concern of education in general and of mathematics education in particular.

The Verbal Problem of Arithmetic.

Many authors (Van Engen, 1963; Herlihy, 1964; Spitzer, 1967) have recommended the use of word problems in arithmetic for the improvement of problem solving skills. Others, such as Dewey (1938), have spoken rather scornfully of the role of verbal problems of arithmetic. "A problem is not a task to be performed like a so-called arithmetic 'problem' in school (p. 108),"

Morton (1938) stated that the operations of addition, subtraction, multiplication and division, should only be taught as a means to an end; the end being the ability to solve problems (p. 454). By 1963 the role assigned to word problems in the arithmetic program had been completely reversed. The proper role of verbal problems, according to Van Engen (1963), was to provide a meaningful physical setting, "for the building of the concepts of an operation (p. 5)."

Obviously some of the verbal problems found in arithmetic text-books would not qualify as 'true' problems in the sense of Dewey's

phases of reflective thought or of Gagné's "higher-order learning." Henderson and Pingry (1953) pointed out that textbook exercises can only be considered "problems" if the student accepts them as his own and their solution becomes his own personal goal (p. 231).

Some authors, however, feel that there is a further condition required for a textbook problem to become the focus of real problem solving activity. The stipulation is that the individual does not possess a habitual, ready-made, response by which the solution may be attained. "If a pupil produces an appropriate response from habit, or sees no relationships in the conditions, there _is_ no problem for him (Herlihy, 1964, p. 308)."

The kind of problems which should be posed in an arithmetic classroom has also been discussed recently. Avital and Shettleworth (1968) recommend the use of "higher-order" problems. An essential characteristic of this type of problem is that it require the non-routine manipulation of previously learned material and the discovery of relationships among previously unrelated concepts and propositions (p. 19).

In summary, we may conclude that problem solving involves some form of reflective thought, a felt need and an element of uncertainty, the formulation of hypotheses and the selection of an appropriate response chosen on the basis of deductive reasoning or empirical evidence, and some form of action based on that choice. It may also involve the application of previously learned concepts to a new situation in such a way that a new rule or principle is learned. It is only when textbook problems of arithmetic meet some or all of these conditions and elicit from students some form of non-habitual, choice-

making behavior, that such problems can be considered as contributing to the growth of problem-solving ability. It follows that the usual tests of arithmetic problem solving, ordinarily used in the elementary classroom to measure achievement in problem solving, cannot be considered a reliable indication of a student's capacity to apply previous learnings to new situations. This conclusion has important implications for the design of the present study.

II. THEORIES OF INQUIRY AND DISCOVERY LEARNING

Discovery teaching, or the inquiry method, has been widely recommended to teachers, particularly of science or mathematics, for well over a decade (Swenson, 1954; Bruner, 1961; Kersh, 1964; Suchman, 1966). The merit of this method seems to lie in the belief that it is natural for children to inquire, and that when they are engaged in inquiry they are learning strategies which will guide their thinking and make them want to learn all through life.

Kersh (1964) distinguished among "independent" discovery, "guided" discovery and "directed" learning. He believes that the benefit to be derived from learning by discovery comes from the fact that the learner may engage in greater amounts of practice in employing problem solving strategies and in making applications than he would by some other teaching-learning process. Independent discovery, claims Kersh, has a "mystical" motivating power that is unique to discovery learning. Guided discovery, however, is necessary to exercise and reinforce the learner in "searching" behavior: strategies of problem solving, divergent as opposed to convergent thinking, and flexibility (p. 230).

Much of the impetus for inquiry learning has come indirectly

from the developmental psychology of Piaget and his concept of equilibration: i.e. the child assimilates cues from his environment and accommodates his existing mental structures accordingly. At the Cornell Conference in 1964, Piaget was quoted as saying:

> The accent must be on auto-regulation, on active assimilation - the accent must be on the activity of the subject. Failing this there is no possible didactic or pedagogy which significantly transforms the subject (Ripple and Rockcastle, 1964).

The following statements, made at the same conference, have also been attributed to Piaget:

> The goal in education is not to increase the amount of knowledge, but to create the possibilities for a child to invent and discover. . . . Teaching means creating situations where structures can be discovered; it does not mean transmitting structures which may be assimilated at nothing other than a verbal level . . . . A teacher would do better not to correct a child's schema, but to provide situations so he will correct himself (Duckworth, 1964, p. 3-4).

One of the problems, it appears, with discovery or inquiry learning, is that there are so many different interpretations of how it should be carried on in the classroom. Henderson (1957) gave this formula for using the discovery method:

1. The teacher has in mind a generalization he wants his students to learn.
2. The teacher selects instances of the generalization for study.
3. The teacher directs the pupils' thinking relative to the instances by suggestions and questions.
4. The teacher educes, if possible, the statement of the generalization from the student (p. 288).

Obviously, Henderson has described the inductive method of instruction which most teachers use at least some of the time and which Kersh might call "guided" discovery.

Suchman (1964, 1966) has given a rather precise procedure for what he calls an "inquiry session," usually in the context of a science lesson. He also outlines three conditions which he states are necessary for inquiry to occur in classroom settings. These are: (1) a focus

for attention; (2) freedom to reach out for data; and (3) a responsive environment (1964, p. 105-106).

While Suchman's description of discovery is probably closer to Kersh (1964) and to Piagetian psychology than that of Henderson, the activity methodology described by Davis (1964) and Biggs and MacLean (1969) might be more representative of Kersh's "independent" discovery. In mathematics particularly, the activity or inquiry method of teaching has come to mean allowing children the freedom to explore mathematical ideas and to discover patterns and relationships for themselves.

Davis (1964) describes two types of student activities which are useful in helping children to inquire: those in which they gain experience by doing something - such as guessing the size of an angle and checking their guesses by measurement; and those in which a group of children pursue a seminar-type discussion, unobtrusively guided by a teacher. Davis reports that these informal exploratory experiences are characterized by lively student participation, a relative absence of exposition by the teacher, a relatively non-authoritarian tone, an absence of external rewards and an unusually low anxiety level (p. 135).

While Bruner (1961) and others have suggested that education may be on the "threshold of a renaissance" because of the increased use of the discovery method, not all are in agreement. Cronbach (1966), in a critical appraisal of learning by discovery, cited the need to study the five-fold interaction among subject matter, instructional type, timing, type of pupil, and desired outcomes. He suggests that discovery teaching might not be the best method to use with all pupils all of the time.

In agreement with this view is another critic of the current

"discovery" trend. Ausubel (1964) has made these statements with regard to discovery:

1. Learning by discovery has its place among the repertoire of accepted techniques available to teachers but it should not be considered a panacea; hence it is not a question of whether it should be used in the classroom, but rather for what purposes and under what conditions.

2. All problem solving and laboratory experience is not necessarily meaningful.

3. The discovery method has the psychological limitation that it is generally inappropriate for teaching subject matter content, except when pupils are in the <u>concrete stage of cognitive development</u>.

4. Students who possess a sound meaningful grasp of rudiments of a discipline like mathematics, can be taught this subject meaningfully and with maximal efficiency, through the method of verbal exposition, supplemented by appropriate problem solving experience.

5. Simply on a time-cost basis, if secondary-school and university students were obliged to discover for themselves every concept and principle in the syllabus, they would never get much beyond the rudiments of any discipline (pp. 231-260).

As critical as Ausubel seems to be of the trend towards discovery methodology, he does however admit that it is "indispensable for teaching scientific method and effective problem solving skills (p. 230)."

In this section we have examined various interpretations of the theory of discovery, inquiry or activity learning. While some authors have recommended a complete acceptance of these methodologies, and along

with them sweeping changes in school organization and curriculum, others have grave reservations. All seem to agree however, that discovery and inquiry are necessary for the development of problem solving ability and that discovery teaching is particularly appropriate when children are in the stage of concrete operations.

III. THE ROLE OF LANGUAGE IN PROBLEM SOLVING

Many studies have pointed to the fact that verbal facility plays an important part in the process of problem solving, particularly those verbal skills related to the formulation of hypotheses. Martin (1963) reports a study which provides evidence of the extent to which the higher-order verbal processes, as measured by tests of reading comprehension and of abstract verbal ability, are important in determining success in arithmetic problem solving.

> A basic facility in arithmetic computation is necessary in problem solving, but when computational steps are placed in a verbal context certain additional complex abilities are required. Moreover, this appears to be more than skill in reading *per se* (p. 4548).

It does not appear that studies have been made to examine how the ability to use language, as distinct from ability to read, might be related to ability to solve problems, in particular mathematical problems. And yet, the relationship of language and thought, especially in the cognitive development of children, has received its share of attention in the literature.

Piaget (1926) sees thought as developing from non-verbal, autistic thought, through egocentric thought and speech, to socialized speech which becomes internalized and leads eventually to logical thinking. He looks at the relationship between language and thought

in terms of concrete operations and decides that, while language cannot account for the formation of these operations, "language indefinitely extends their power (1967, p. 93)."

> . . . the more the structures of thought are refined, the more language is necessary for the achievement of this elaboration. Language is thus a necessary but not a sufficient condition for the construction of logical operations (p. 98).

Language development, for Piaget, is a social process achieved through social cooperation and is necessary for the development of logical thinking and for problem solving.

> Out of objectively conducted discussions emerges internalized discussions, or deliberation and reflection. Children learn that there is a morality of thinking: i.e. there are certain rules to be followed. Some of them relate to the avoidance of contradiction, the need for proof, and the obligation to keep constant the meanings of words. As does operational thought, social cooperation involves systems of operations and is a strong force in the development of logical operations (1960, p. 166).

Vygotsky (1962) assigns an important role to egocentric speech in the child. He believes that it becomes internalized in the form of verbal thought and that it "increasingly serves problem solving and planning as the child's activities grow more complex (p. 16)." He agrees with Piaget that the child masters the syntax of speech before he masters syntax of thought:

> Piaget's studies proved that grammar develops before logic and that the child learns relatively late the mental operations corresponding to the verbal forms he has been using for a long time (p. 46).

Elkind (1967) makes a distinction between thought and language: thought originates with the child as it is the result of his activity; language is derived from imitations of patterns provided by others. However, "once language appears, it is in constant interaction with thought . . . ." He concludes that, with the aid of language, thought

can do much more than it would otherwise be capable of doing (p. xvi).

Bruner (1958) also attributes an important role to language in information-processing which he believes is basic to concept formation and to problem-solving.

> Words tend to direct attention to the attributes they signify. If there are no appropriate words, then there is a tendency to fail to perceive cues that could have status as attributes (p. 40).

Bruner's (1966) theory of instruction emphasizes the importance of language development in the process of education:

> If there is not a developed awareness of the different functions that language serves, the resulting affliction will be not only lopsided speaking and writing but a lopsided mind. . . the afflicted person will be restricted in his coping (p. 109).

The idea that deficiency in the use of language is usually accompanied by deficiency in thought has been expressed by several other authors. Loban (1963) suggested that there might be a relationship between tentative thought and language achievement in elementary school children. He reported that those subjects who had the greatest power over language, "by every measure that could be applied," consistently used language to express tentativeness; supposition, hypotheses and conditional statements, verbal skills essential to the process of problem solving, occurred much less frequently in the language of subjects lacking skill in language.

> The child with less power over language appears to be less flexible in his thinking, is not often capable of seeing more than one alternative, and apparently summons up all his linguistic resources merely to make a flat dogmatic statement (p. 54).

He concluded that the expression of tentativeness appeared to be a function of language "which distinguished effective and ineffective users of language (p. 53)."

One obvious difficulty in testing Loban's hypothesis is that

the evidence of tentativeness on the part of a subject would most likely be governed largely by the stimulus situation: i.e. the manner in which the language sample to be examined was gathered would probably determine whether a subject would use conditionals and suppositions. There would also be the real difficulty of distinguishing between indecision or uncertainty expressed by tentative statements such as "it might" or "maybe" and real tentative thinking such as that encountered in the formulation and testing of hypotheses.

Loban's findings do, however, seem to be consistent with Bernstein's (1967) theory that children from a low socio-economic background operate linguistically within a restricted verbal planning function and have few strategies available to them to generalize or to order a verbally-presented arithmetic problem. Bernstein suggests that such a child would frequently use statements in which reason and conclusion are confounded to produce a categoric statement (p. 94). Bernstein's restricted language users and Loban's low language achievers seem to display a low tolerance of ambiguity and a certain inability to suspend judgment. Both of these characteristics, however, would appear to be essential to tentative thought and consequently to efficient problem solving.

The work of Tate and Stanier (1964) points to a relationship between the ability to tolerate ambiguity and success in problem solving. They reported that poor problem solvers, when they did not know a correct answer to a problem and were faced with responding true, false, or not enough facts, tended to reject the last choice and to answer either true or false. Good problem solvers, on the other hand, seemed to be able to accept the fact of not knowing for sure and to choose the

the response "not enough facts" if they were uncertain about a correct answer.

Halliday (1969) agrees that education needs to develop in children an awareness of the differenct functions that language serves. He suggests that the child who operates linguistically within a "restricted code" may be deficient in the range of uses which he perceives for language, in particular the heuristic function of language: i.e. the use of language to explore or the function of "tell me why." He concludes that, in order to be taught successfully, it is necessary to know how to use language to learn and to "participate _as an individual_ in the learning situation (p. 35)."

In summary, it seems that while language cannot be said to determine the child's experience, it certainly serves a mediating function in determining the individual's cognitive processes, the development of logical thinking, and the ability to engage in problem solving. It appears entirely possible that while the components of tentative thinking - the formulating of hypotheses, predicting and validating, being prepared to reject one alternative and search for another - are skills which are required for "higher-order" problem solving, these may not be within the grasp of the child who is deficient or restricted in the uses he perceives of language.

IV. RELATED RESEARCH

The Processes of Problem Solving

Attempts to analyze the processes used by individuals as they engage in solving a problem are by no means a recent development. Monroe (1928) surveyed over 9,000 students in grades six, seven and

eight to examine their approaches to the solution of verbal problems of arithmetic. He summarized his findings as follows:

> Although the data of this investigation are not entirely consistent, they appear to substantiate the conclusion that a large per cent of seventh-grade pupils do not reason in attempting to solve arithmetic problems. Relatively few pupils follow formal analysis procedures. Instead, many of them appear to perform almost random calculations upon the numbers given. When they do solve a problem correctly, the response seems to be determined largely by habit. If the problem is stated in the terminology with which they are familiar and if there are no irrelevant data, their response is likely to be correct. On the other hand, if the problem is expressed in unfamiliar terminology, or if it is a 'new' one, relatively few pupils appear to attempt to reason. They either do not attempt to solve it or else give an incorrect solution (p. 19).

It would be reasonable to conclude that many of Monroe's subjects were not really engaged in problem solving, at least not in the sense of the process of problem solving as defined by Henderson and Pingry (1953).

Lazerte (1933) investigated the mental processes of pupils from grades two to seven as they attempted to solve problems in arithmetic. He used a rather ingenious method, called the Envelope Test, involving a sequence of choices of alternatives made by a subject in the course of trying to solve a problem. Some of his findings indicated that: (1) there is much trial and error procedure in problem solving whenever the problem is relatively difficult; (2) pupils vary greatly in the flexibility of their methods and in the amount of experimentation in which they engage; (3) pupils who are deficient in reading ability are handicapped in problem solving; and (4) problem solving ability depends upon many factors (pp. 123-126).

While several psychologists have devised techniques for studying problem solving in the laboratory, Duncker (in Henderson and

Pingry, 1953) was one of the few who used the "thinking-aloud" technique to study how complex mathematical problems are solved. He confirmed the effect of set on the process of problem solving, noting that it affected the search model, narrowed the range of the search, inhibited the perception of certain relationships and blocked the formulation of certain hypotheses, with the result that not all relevant thought material was made available (p. 243).

Corle (1958) studied the thought processes of sixth-grade students while solving arithmetic problems and attempted to determine the importance of several factors: methods of reasoning, confidence, and an understanding of vocabulary. Making use of the individual interview technique, Corle had seventy-four pupils solve eight problems. The following implications were drawn from his findings:

1. A pupil's idea of what a problem means is important to him. . . .
2. Word and number clues served as the predominant method of attack. More than five out of six problems were approached in this manner.
3. Pupils tend to be overconfident anout their arithmetical success. This undue assurance indicates over-dependence upon computation as a tool for problem solving.
4. Understanding the terms used in arithmetic is a definite factor in problem solving efficiency. . . .
5. The greatest number of incorrect solutions occurred in problems where the method of solution was not apparent, but was dependent upon deductions based on experiences with the idea. . . .
6. The real reasons for missing most of the problems were not computational ones. (p. 203)

Using the same interview technique, Miller (1960) analyzed the thinking of sixth-grade pupils in regard to their solution of seven verbal problems. He found that overt analytical responses seemed to be more closely associated with correct solutions than were overt computational responses.

An investigation by Brian (1966) led him to identify four

processes of mathematical problem solving: (1) the constructing of models (diagrams, etc.); (2) the process of conjecturing; (3) the settling of conjectures as either true or false; and (4) using or applying known or given axioms or theorems or algorisms to solve problems.

Believing that traditional programs in mathematics are designed primarily towards having learners acquire the fourth process, Brian designed a four-week treatment for college students to encourage the use of the first three processes. The treatment involved the use of behavioral flow diagrams - a conjecture search loop and a conjecture verification loop. Results suggested that the subjects did acquire some process behavior they had not previously possessed, particularly with respect to the process of settling conjectures, which showed statistically significant gains. Brian concluded that instructional programs could be designed to assist students to acquire these processes and that guessing and intuition can play important roles in problem solving when a known method of approach is not readily available.

In 1967, Gorman wrote a comprehensive critical analysis of research on written problems in elementary school mathematics in which he examined 293 studies on problem solving, rejecting all but thirty-seven of them on the basis of poor internal or external validity. He reported many conflicting results, even among the accepted studies, which led him to conclude that methods used by students to solve problems needed additional analysis with a focus on the thinking process during the solution of verbal problems. In particular, the "manner in which one can monitor the process of thinking during problem solving is in need of additional exploration (p. 277)."

Kilpatrick (1967) expressed the opinion that the "thinking aloud" technique was the best one available at present for getting a subject to produce sequentially-linked, and at the same time observable, behavior as he engages in problem solving. He is also convinced that such analyses of behavior in connection with classroom problems of arithmetic are urgently needed at the present time.

> To make our knowledge of problem solving behavior relevant to education, we must eventually study how students solve problems of the sort they meet in the classroom. If at present there are too many uncontrollable and even unknown sources of variation for careful experimental studies, we should at least attempt analyses of behavior. Such analyses, though they run a high risk of producing little immediate payoff, are necessary for directing future experimentation (p. 2).

Kilpatrick pointed out that whereas introspection and retrospection, sometimes used in analysing behavior during problem solving, require an <u>analysis</u> of the composition of thought, thinking aloud requires only that the subject give an <u>account</u> of the activity of his thought. That is, "he reports mental acts rather than analyzing mental states (p. 7)." A further advantage of thinking aloud as a technique for investigating the processes of problem solving is that it permits lengthy observations and the use of subjects without special training. They do not have to think aloud and observe themselves thinking at the same time.

On the other hand, Kilpatrick recognized that the method has its disadvantages. Thinking out loud may inhibit speech or else it may be so rapid and erratic that speech cannot follow it or state it precisely. It is also possible that subjects may remain silent just when their thought seems to be most active, requiring the examiner to engage in a certain amount of inference from behavior that is non-verbal. A

more serious limitation is the possibility that a subject may go about solving a problem differently when asked to vocalize his thoughts than he does when left to work at it silently. Several studies (Hafner, 1957; Gagne and Smith, 1962; Roth, 1966) of the effects of overt verbalization on problem solving have found no significant differences on correct solutions, solution time, or modes of inquiry between subjects required to think aloud and those who were permitted to remain silent. Kilpatrick concluded that the risk has to be taken if we are to increase our knowledge of how children solve problems. Consequently he carried out an investigation based on the technique.

Fifty-six subjects, girls and boys, above average in intelligence and who had just completed the eighth grade, were asked by Kilpatrick to think aloud as they attempted to solve a battery of mathematical problems. Their tape-recorded protocols were examined for modes of preparation, production and evaluation, and regularities and similarities noted. The subjects were then divided into groups characterized by similar methods of approach and compared on various measures. Although the use of trial and error was widespread, successive approximation was much less frequently found. Both techniques, however, were related to successful problem solving.

Kilpatrick reported that there was a tendency for more boys than girls to use a trial and error approach and for more girls than boys to use equations. This result may however be attributed, in part at least, to the fact that a preponderance of girls in the sample came from the one classroom where the teacher had taught her class some algebra. This last finding led Kilpatrick to suggest that the sources

of difference in problem solving mode may be traced to one of the important neglected variables in his study, namely, "what goes on in the classroom (p. 96)." Gorman (1967) had also suggested that one important reason why methods research has been unsuccessful might be because "teacher influence" is largely neglected in research design.

The results of Kilpatrick's study appear to support several assumptions with regard to problem solving and its investigation in the classroom upon which the present study is largely based: (1) problems from the classroom are capable of investigation on their own terms; (2) knowledge of problem solving will be enhanced as we learn more about stable individual approaches to problems; and (3) the analysis of problem solving behavior is fully as worthwhile as its manipulation (p. 3).

Methods of Improving Problem Solving

Although the improvement of verbal problem solving in arithmetic has been the goal of numerous research workers for almost half a century, success in such endeavors has, for the most part, been open to question (Gorman, 1967). In 1924, Stevenson reported that:

> It has been shown that pupils' ability to solve problems can be increased materially by consciously following several different methods of instruction. More detailed research should be undertaken to show how much time and effort, if any, should be expended in giving pupils a method of attacking problems, in training them to estimate answers, in assisting them to understand technical words, and other phases of instruction (p. 270).

In 1944, however, Johnson reviewed the literature on problem solving in arithmetic and concluded that research had not shown that "teaching a method of problem solving improves ability to solve problems (p. 482)."

More recently, Pace (1960) attempted to determine the effect of understanding of the processes of problem solving upon problem solving ability. Two groups solved twenty-four specially prepared problem sets over an eight-week period with only one of the groups engaging in class discussions of the "hows" and "whys" of the solutions. Using two different forms of the Arithmetic Reasoning Test of the Stanford Achievement Battery for the pre-test and post-test, Pace found that the non-discussion group made negligible gains, while the discussion group made statistically significant gains. She made three suggestions for the improvement of problem solving: (1) provide the children with opportunities to solve many problems; (2) encourage the use of multiple solutions of problems; and (3) provide for the development of the understanding of the four fundamental processes of arithmetic.

Riedesel (1964) studied the effect on problem solving ability of providing practice in solving problems which are appropriate in level of difficulty for each pupil in a class. Although his results show significant gains for the experimental group, they are clouded by the fact that in addition to providing for two levels of difficulty he also imposed specific techniques for assisting students to solve the problems. It is difficult to determine which of the two variables was responsible for the improvement, the appropriate level of difficulty, or the specific techniques.

Post (1968) studied the effects of the presentation of a structure for problem solving upon success in problem solving in grade seven mathematics. He concluded that: (1) special study of problem solving appears not to enhance problem solving ability; (2) intelligence is a significant factor in the determination of problem solving ability;

(3) creativity is not a significant factor; and (4) there are no significant sex differences in problem solving ability.

While the inquiry or discovery method of teaching is claimed to assist the development of problem solving ability, there exists little actual research to support the claim. Most of the literature on activity or discovery learning can be characterized as theoretical discussions or descriptions of developmental efforts (Kieren, 1969). Studies which have been made, however, have yielded conflicting results such as those reported by Scandura (1968) when he attempted to identify the relative value of the hypothetical (discovery) mode and the expository (direct) mode of problem-solving instruction.

Wills (1967) investigated the effect of learning by discovery on the transfer of problem solving ability. He found that the experimental groups doubled their pre-test performance whereas a control group who did not receive instruction in looking for patterns which helped them to discover a generalization, made only minor gains. The scope of his experiment was limited by the fact that the instructional materials designed to guide subjects in discovering a generalization were only used by the experimental group for a two-week period.

Larson (1964) investigated the strategies of inquiry of four groups of fifth-grade students. Of the two groups classified as experimental, one had had experience in classroom inquiry over a two-year period, the other for only one year, but with the same teacher. The groups were roughly comparable in intelligence, distribution of sex, and age. Each child in the sample was given four problems consecutively in an interview-type situation. Two problems characteristic of Piaget's studies of logical thinking and two problems characteristic of Bruner's

studeies of children's strategies were used. The findings of the study were somewhat equivocal because the strategies of the group with <u>two</u> years experience in classroom inquiry proved to be the most sophisticated of the four groups; but those of the group with only one year of experience in classroom inquiry were less sophisticated than even the two control groups. The major conclusion of the study was that the four components of the strategies (objectives, organization, systematization and validation) could provide dimensions for futher studies.

In 1969, Slinn made a study of teacher influence on the inquiry skills of grade six students and found that there was a significant difference in achievement on a test of scientific inquiry by pupils of teachers who were classified as "indirect" compared to the achievement on the same test of the pupils of a "direct" teacher. Although neither the Larson nor the Slinn study were directly concerned with arithmetic problem solving, it seems that both may have implications for the effect of inquiry teaching on problem solving strategies.

What emerges from this review of the research into the effects of various teaching techniques or methods for improving problem solving ability is that there <u>is</u> no one "method" for solving problems since each problem must determine the method. Most of the studies tend to support the thesis that problem solving is not a skill which may be acquired by a mechanical or sterotyped procedure and that teaching techniques pertaining to discovery or inquiry methods are in need of further exploration. Like Van Engen (1963) we are led to conclude that the relative superiority of any one "method" over another is extremely doubtful and that the effectiveness of a particular method probably depends upon the zeal and skill of the teacher using it (p. 403).

V. SUMMARY

The first three sections of this chapter were devoted to a discussion of the nature of problem solving, of inquiry and discovery learning and of the relationship of language, tentative thinking and problem solving. Three important points emerged from the discussion.

Firstly, problem solving implies personal involvement in search behavior. Ideally, for each problem posed, three things should occur after the individual has become aware of the problem as something he would like to try and solve: he defines the problem or <u>senses</u> the situation or conditions imposed by it; he identifies various hypotheses (<u>predicts</u>) that could lead to a solution; and he <u>verifies</u> or tests the hypotheses for feasibility. Verbal problems of arithmetic may or may not be real problems inasmuch as they evoke or fail to evoke this type of behavior.

Secondly, inquiry or discovery teaching may facilitate the development of problem solving ability in students who are exposed to this kind of teaching. It is probably most suitable for children who are in the concrete stage of logical development and who have not yet acquired the mental structures required for real logical thinking.

Thirdly, language or verbal skills are an important part of the problem solving process. Efficient strategies of problem solving, such as considering various hypotheses and engaging in tentative thinking, may require an awareness of a particular function of language - the heuristic, or "tell me why" function.

The final section of the chapter contained a review of related research with regard to the processes of problem solving and the effects

of various techniques or instructional methods for improving problem solving. Many factors were found to contribute to successful problem solving: age, intelligence, persistence, computational facility, type of problem, motivation, reading comprehension and a knowledge of vocabulary, as well as the ability to vary one's strategies and to think in a hypothetical mode while searching for a solution to a problem. Teaching methods designed to assist children to develop a flexible approach to problems should lead to improved problem solving ability.

CHAPTER THREE

DESIGN OF THE INVESTIGATION

The purpose of the investigation was two-fold: (1) to investigate the effectiveness of an inquiry approach to the teaching of arithmetic in the development of flexible and efficient strategies for solving mathematical problems; (2) to examine the relationship of ability to use language effectively, the degree of tentative thinking which a student exhibits while seeking a solution to a problem, and successful problem solving. This chapter outlines the procedures which were followed in choosing the population from which the sample of students was drawn. It contains a discussion of the arithmetic program in the classrooms which were designated as either Inquiry or Non-Inquiry, a description of the instrumentation and of the procedures used to analyze the results of the study.

I. POPULATION

Selection of the Participating Classes

The first step in choosing suitable subjects for the investigation led to the problem of identifying classrooms in which teachers were using an 'inquiry' approach to the teaching of arithmetic and those which were not using this method of teaching. A questionnaire, designed to assist in this identification, was developed and sent to the principals of ten schools for distribution to teachers of grade six arithmetic. The schools had been randomly chosen from among those of the Public School System in Edmonton, Alberta. The questionnaire was followed up with a visit by the investigator to these ten schools and

a personal interview with each teacher of grade six arithmetic. During the interview, the contents of the questionnaire were discussed and an attempt was made to ascertain to what extent an 'inquiry' or 'non-inquiry' approach was being followed by the teacher and the pupils. Permission to visit the classroom during an arithmetic period was requested and an appointment made. A checklist designed to guide the observer was developed for use during the visit and is included with the questionnaire in Appendix A.

It was possible to contact eighteen teachers in the manner outlined above. Of these, only two seemed clearly to be following an 'inquiry' method of teaching arithmetic and were therefore designated Inquiry teachers. From the remaining sixteen teachers, four were chosen as being representative of sound but somewhat "traditional" teaching and were designated Non-Inquiry.

Assistance was then secured from the Supervisor of Mathematics for the Edmonton Public School Board to identify two more teachers who were using an 'inquiry' approach. This brought the total number of teachers in each group to four. Particulars of training and teaching experience for the teachers who participated in the study are given in Table I and details with regard to the classes in Table II.

The final population from which the sample of children was drawn consisted of eight class units located in seven schools. Two of the classrooms, one Inquiry and one Non-Inquiry, were situated in open-area schools. The other six were self contained. Since socio-economic status was not to be controlled in this study, it was deemed important that the schools which were participating be located in

TABLE I

TEACHERS: TRAINING AND EXPERIENCE

Inquiry		Non-Inquiry	
Yrs/Training	Yrs/Teaching	Yrs/Training	Yrs/Teaching
5	5	6	17
4	2	4	12
5	15	4	30
4	2	3	8

TABLE II

NUMBER OF STUDENTS AND DISTRIBUTION OF SEX IN PARTICIPATING CLASSES

Inquiry				Non-Inquiry			
Class Code	Male	Female	Total	Class Code	Male	Female	Total
Ia	18	13	31	Na	14	14	28
Ib	11	17	28	Nb	15	13	28
Ic	16	17	33	Nc	13	15	28
Id	17	14	31	Nd	15	15	30
Total			123	Total			114

approximately equivalent socio-economic areas. The communities served by the participating schools, while varying widely in geographic location in the city, were designated by school officials as average middle-class communities. None of the areas could be regarded as affluent; neither were they considered low socio-economic or inner-city communities.

Description of the Arithmetic Programs

The classrooms which were designated Inquiry had been differentiated from the Non-Inquiry classes on the basis of the questionnaire, assistance from the Supervisor of Mathematics, and on the basis of observations made during a visit to each classroom. In order to assure a common ground for interpretation of the terms Inquiry and Non-Inquiry, a full description of the arithmetic program in the eight classrooms will be given here.

The Inquiry Classrooms. An inquiry class was identified as one in which the 'textbook' method of teaching grade six arithmetic had been largely discarded, at least since the beginning of the fall term, in favor of either small group investigations or a modified plan for the individualization of instruction through a program of individual inquiry. The emphasis in the Inquiry classes was on mathematics activities often, but not always, involving manipulative materials, with each student proceeding at his own rate and assuming the major portion of the responsibility for his own progress. No formal lessons were presented to the class as a whole. Students were encouraged to "think out" a problem or an activity either by themselves or with one or more of their classmates.

Two of the classrooms designated as Inquiry were using the small

group investigations approach. These "investigations" for the most part were teacher-constructed and consisted of open-ended questions, problems or directions for pursuing a mathematical activity: e.g. "How many times do the hands of a clock form a right angle during one twelve-hour period?" or "Compare your pulse rate before and after exercise. Make a chart or graph to report your findings."

The classes were divided into four groups with seven or eight pupils in each. On four consecutive days the groups rotated among four "stations" as follows:

> <u>Station One</u> - The students began an investigation, tried to think how it should be carried out and findings presented; recorded ideas for discussion and began to collect data.
>
> <u>Station Two</u> - This was a teacher station: problems which had been encountered the previous day were discussed; data was examined and related topics and/or problems discussed; individual assistance was given when necessary by the teacher.
>
> <u>Station Three</u> - Students worked either in pairs or individually on exercises and assignments related to the investigation. (This was essentially a "drill" or skill practice day.)
>
> <u>Station Four</u> - This was "game" day: games and game-like activities were used to reinforce the mathematical ideas related to the investigation just completed.

Every fifth day in a series, the students had what was called a stop day. For half of the period they discussed one or two of the investigations which all had completed. Encouragement was given by the teacher to sharing ideas and discussing why some group's findings differed from others. The remaining time was devoted to the presentation of results of some of the investigations in the form of charts, graphs, or other visual materials. Concrete materials such as stop watches, scales and balances of all kinds, cardboard, beads, string, coloured paper, etc. were readily available to the students in these classrooms.

The two classrooms just described had been designated for the investigator by the supervisor of mathematics because he knew that they were using an 'inquiry' approach. However, when the classrooms were visited in the latter part of April only one of them was continuing to use the "investigations" procedure just outlined. The other teacher reported that he had decided to use "selected pages" from the <u>Seeing Through Arithmetic</u> series "for review purposes". There was ample evidence that the "investigations' approach had been followed for most of the year. Students were proceeding with the "review" pages at their own rate and checking of responses from the teacher's response verification book was carried out as required either by pairs of students or individually. The teacher appeared to be giving assistance to those who required it, but in an unobtrusive manner.

The other two Inquiry classrooms in the study were located in the same school and were following a modified individualized, multi-text approach. The grade six curriculum had been divided by the teachers into eight major sections or "Big Ideas." A pre-test was taken prior to beginning a unit to determine each pupil's strengths and weaknesses in the area. After the pre-test, selected guide sheets were given to the pupil indicating several text-books which the student could consult as well as various related activities that the student was expected to carry out. Students then worked either alone or in pairs, giving each other assistance or asking the teacher for assistance. The teachers occasionally drew a small group together for direct teaching on topics with which these students were experiencing difficulty; but for the most part, students were encouraged to search out materials on their own. The emphasis seemed to be on learning to direct their own

learning activities and to think through problems for themselves.

When a student felt that he had covered a "Big Idea" adequately, he asked the teacher for a "mastery" test. If he scored 80% on the test, he was considered to have achieved mastery and so went on to the next section; if not, then he was asked to try to figure out where his difficulty arose: "Was it a lack of understanding or of practice?" The teacher then assisted in choosing materials and activities for the student to "work through" before he was allowed to take another form of the same mastery test. When the eight "Big Ideas" had been mastered, pupils were allowed to use source material which was available in the classroom to explore any of several related mathematical ideas. About one-third of the class were engaged in this kind of supplementary work at the time of the experimenter's visit.

The Non-Inquiry Classrooms. A non-inquiry classroom was identified as one in which the teacher admitted that he "stuck pretty close to the textbook" and spent less than twenty-five per cent of class time having pupils work with materials or activities other than those contained in the prescribed text. In each case the classes were using the *Seeing Through Arithmetic* series. At the time of the classroom visit, the teacher was engaged in introducing some new mathematical concept, using an approach which could be characterized as "show and practise". A lesson was presented to the entire class, with the teacher giving several instances of a concept and then drawing a generalization or rule for the class. In two of the classes division of fractions was being introduced. The method was explained and then one or more pages of exercises was assigned from the text. As the students worked at these, the teacher went around the room giving individual help where needed.

Many 'good' teaching techniques were in evidence in these Non-Inquiry classrooms: having the more able students teach the slower ones; requiring the drawing of diagrams to represent ratios and fractions; the use of set notation to identify different names for a point on a number line; as well as many motivational devices.

While teaching methods obviously differed widely among the four classrooms, they all had this in common: a dependence upon the prescribed text as a teaching tool. The emphasis seemed to be on how many exercises a student could do without making an error. Teachers admitted that the students didn't want to work independently, that "they like to be directed," or else that the curriculum was too heavy to permit much time for supplementary activities of an exploratory nature. The approach to the teaching of problem solving was to assign a set of problem exercises after teaching a method or an algorithm which would lead to their solution. Pupils asked fewer questions than they did in Inquiry classrooms, engaged less in student-initiated activities and seemed to expect to be shown "how" to do something rather than attempt to do it on their own initiative.

II. THE SAMPLE

From each of the eight participating classes, a sample of ten subjects was chosen, using a table of random numbers (Keeping, 1962). In order that randomness might be preserved, no attempt was made to stratify on the basis of sex. However, in every subgroup of ten, it happened that the distribution of sex was either a five-five or a four-six split. The total number of subjects in the sample was thus eighty, forty in the Inquiry group and forty in the Non-Inquiry group.

The following reasons were accepted for the elimination of a subject from the sample: absence from school on the testing days; a severe learning or emotional disability; recent transfer to the school or the classroom; difficulty with oral communication due to a language barrier or physical disability such as deafness. In all, six students were eliminated and replacements made on a random basis.

While it had been assumed that randomization of the sample would ensure representativeness, the mean total I.Q. for each group of ten subjects within a class was compared to the mean of the class from which the subsample was drawn. These comparisons are shown in Table III. It will be noted that, while the means for the subsamples of ten were sometimes higher or lower than the class mean, the mean I.Q. of the Inquiry group was almost identical to the mean of the Non-Inquiry group. Using a Scheffé test to compare the means of the classroom subsamples, it was determined that in no case did differences among means approach significance.

TABLE III

MEAN I.Q. FOR SUBSAMPLE OF TEN AND FOR CLASS GROUP

Inquiry			Non-Inquiry		
Class Code	Subsample	Class	Class Code	Subsample	Class
Ia	108.5	108.6	Na	110.7	105.8
Ib	109.9	105.5	Nb	109.5	108.6
Ic	106.9	104.2	Nc	106.3	102.6
Id	109.4	104.4	Nd	108.5	111.8
Total	108.5	105.2	Total	108.7	107.2

III. INSTRUMENTATION

Standardized Instruments

Each pupil's intelligence was measured by the Canadian Lorge Thorndike Tests of Intelligence (Level 3). Both Verbal and Non-Verbal Batteries were administered and scored during the month of January, 1970, by the classroom teachers as part of a system-wide testing program. The intelligence quotients calculated for each subject, as recorded in the cumulative records, were used for this study.

The reliability of these tests of intelligence has been well established. Alternate forms have been found to correlate from .76 to .90 at all level of the tests. Most reviewers agree that the Lorge Thorndike Tests occupy a place among the best of the group intelligence tests and the uses recommended for them are both reasonable and defensible (Buros, 1959, pp. 480-482).

Since the study was intended to examine the relationship between achievement on a conventional arithmetic problem solving test and problem solving ability (measured by the Strategies of Problem Solving Test), it was necessary to choose a standardized test of arithmetic problems. The Iowa Tests of Basic Skills Arithmetic Problem Solving Subtest A-2 (Form 3) was chosen for this purpose and administered to all the students in the participating classrooms by the investigator. Directions for administering the test were carefully standardized (Appendix B) and the thirty-minute time limit was closely adhered to.

Herrick reports that the reliability coefficients of the subtests of the Iowa battery are sufficiently high for individual diagnosis and prediction. He warns, however, that intercorrelations among the subtests

indicate a heavy loading of all the subtests with vocabulary and reading skills. The intercorrelations suggest that not much would be left after the effect of vocabulary and reading skills was removed (Buros, 1959, p. 32).

<u>Rating</u>: <u>Language Achievement</u>

Language achievement was considered an important variable in this study but a reliable instrument to measure it proved difficult to find. Since Loban (1963) had reported that children who had the greatest power over language, by "every measure" that could be applied, were more likely to use tentative expressions, it was believed that any one of several devices for measuring achievement in language would be satisfactory.

Several alternatives were considered, among them the Writing Test of the <u>Sequential Tests of Educational Progress</u>. However several reviewers of the test warned that it was not a very satisfactory instrument. Jackson (in Buros, 1959, p. 64) wrote:

> The so-called "writing" test is not very satisfactorily named . . . . (It) does not actually involve any writing but consists rather of a critical evaluation of what others have written.

The idea of using the S T E P Essay Test was also considered and again discarded since reviews caution against the large element of subjectivity in the scoring of the test. Jackson concluded that the test did not represent much improvement, if any, over teacher-assigned topics and teacher-scored examinations (p. 65).

It was finally decided to use a combined language rating obtained from two sources: (1) the classroom teacher's rating of a subject's oral and written language competency and (2) a score

assigned by the investigator on the basis of an original composition entitled Something I Like and Why I Like It which was submitted by each subject. The scoring of these written assignments was based on criteria similar to those outlined by Coutts and Baker (1955). While recognizing the limitations of these criteria (Appendix B), it was hypothesized that they would be typical of what teachers actually use when rating a student's composition. The final score assigned to each subject for language achievement was based on a five-point scale, with five indicating superior achievement and one indicative of very low achievement. Whenever there was a major discrepancy between the classroom teacher's rating and the grade assigned for the written composition, one or more competent judges at the University of Alberta were consulted.

Rating: Tentativeness

Since the investigator had hypothesized that tentativeness in the language of the subjects would be exhibited to a greater degree by those students who were considered high language achievers and by those who used the most efficient problem solving strategies, it was necessary to choose some means of measuring tentativeness. Two of the problems from the Strategies of Problem Solving Test seemed most likely to elicit expressions of tentativeness on the part of the subjects. Consequently, the tape-recorded protocols for these two problems (Appendix C) were examined and evaluated by the experimenter on the basis of whether they provided evidence of tentative thinking on the part of the subject. In this manner each subject was identified as being either tentative or non-tentative for each of the two problems.

Since the scoring of the tentativeness measure involved

a subjective analysis, it is important that the meaning of the term be well established. Tentativeness, as the term is used in this study, is not to be confused with indecision or uncertainty as to how the problem should be solved; it is associated with a genuine search for a method of proceeding that will yield a verifiable answer, with a willingness to discard a solution that cannot be verified or that is obviously inappropriate, and to consider one or more alternative. The following example illustrates tentativeness in connection with Problem III (how much copper):

Subject's comments	Experimenter's comments
Would it be 72 pounds?	You think they would need 72 lbs. of copper to make 24 lbs. of the coins.
No, that would be too much. Let's see... the 24 lbs. is nickel and copper both is it?	The coins are made from both nickel and copper.
And it's got to be 1 pound of nickel to 3 lbs. of copper?	Yes, that's the recipe.
If I times them both by 4... no, that wouldn't be enough. Let's see, could it be 5 of nickel and 15 of copper? No, that's not enough either. I've got it now!	You think you've got it.
Yes. It's 6 of nickel and 18 of copper because 6 times 3 is 18 and then 18 plus 6 makes 24 lbs. of the mixture.	So, are you quite sure you have reached the answer?
Yes, because it all adds up to 24 and it's in the right proportion.	I see.

Clearly, in the above example, the subject is using language to hypothesize and to predict as he considers various alternatives in his search for a verifiable solution. While there is nothing "tentative" about his final answer, his earlier trial and error approach involved considerable flexibility and tentativeness. On the other hand, a subject classified as non-tentative would answer the same problem

in a manner illustrated in the following protocol:

I think it would need 72 lbs.	You think that it would require 72 lbs. of copper to make 24 lbs. of the coins.
Well (subject hesitates), it does seem like a lot of copper; but it has to be 3 times as much copper as nickel, so I guess it would have to be 72 pounds.	Do you think there might be some other way of figuring it out?
No, all you can do is just multiply by three, so it has to be 72 pounds.	I see.

While the second subject obviously doubts that 72 pounds is the correct answer to the problem, just as the first subject did, he seems to lack the flexibility required to consider other alternatives. He ends the protocol by making a dogmatic statement that he himself seems to realize cannot be verified. The statements "I think" or "I guess" do not in this instance indicate tentativeness but rather indecision or uncertainty about his response. Several examples of tentative and non-tentative protocols for both Problem III and Problem VII are given in Appendix E.

<u>Strategies of Problem Solving Test</u>

One of the major purposes of this study was to measure and compare the relative efficiency of the strategies used by the subjects when they were required to seek a solution to a problem for which they had not learned an algorithmic procedure. A review of the literature and an examination of existing arithmetic problem tests (such as the <u>Iowa</u> subtest) led to the conclusion that a written test consisting of the usual arithmetic word problems would not be an adequate measure of the processes by which the subjects arrived at a solution.

It was decided to construct a <u>Strategies of Problem Solving</u>

Test, consisting of interesting mathematical problems, but not of the type that grade six students usually encounter in their textbook. The test was given in an oral setting so as to minimize the effect of reading ability and so that an attempt could be made to analyze the thinking processes of the subjects during the problem solving encounter.

A full discussion of the test, how it was constructed, administered and scored is contained in Chapter Four.

IV. DATA COLLECTION

After the population had been identified and the sample chosen, instructions were sent to the participating teachers concerning the written language assignment (Appendix B). The compositions were read and scored by the investigator prior to the testing in the schools.

All testing took place between April 15 and May 12, 1970. The Iowa Problem Solving Subtest was administered during the morning of the first day of testing in each school, following which the ten subjects in the subsample were interviewed and the Strategies Test administered. The testing took approximately two days for each classroom in the study. The tape recordings of the interviews were transcribed by the experimenter within a few days of the test administration.

V. DATA ANALYSIS

Data pertaining to each subject's sex, age, intelligence, test scores, language and tentativeness rating were entered on cards for programming on the 360/67 computer of the Division of Educational Research Services at the University of Alberta in Edmonton.

Hypotheses 1, 2, and 4 were tested by Multiple Linear Regression Analysis. An analysis of the differences in means for several variables for the Inquiry and Non-Inquiry group indicated that age, intelligence and problem solving achievement should be controlled for when testing Hypothesis 1. Intercorrelations were calculated for all the variables in the study using a DESTO 2 program and results were used to test Hypothesis 3. Correlations were deemed to be significant at the .05 level of probability.

The results of the statistical analysis are contained in Chapter Five.

CHAPTER FOUR

STRATEGIES OF PROBLEM SOLVING TEST

A major problem in this study was to find a way of comparing the problem solving ability (strategies) of subjects in the Inquiry group and of subjects in the Non-Inquiry group. A review of the literature on problem solving led to the conclusion that special attention should be given to the construction, administration and scoring of a test that would measure the relative efficiency of the strategies used by the subjects to solve 'real' problems. The present chapter is devoted to a discussion of the test.

I. CONSTRUCTION OF THE TEST

A study by Larson (1966) of the strategies of inquiry used by fifth-grade children when confronted with a series of four problems in an interview setting, served as a model and major inspiration for the design of the present study and for the construction of a test to measure the strategies used by the subjects. However, since the problems used by Larson were not arithmetical, a different set of criteria had to be established to guide the choice of problems for the test. Consequently, the following guidelines were established for the selection of problems:

1. The situations described by the problems should be within the experience of the average child in grade six.

2. They should be of a quantitative nature, but should also prove interesting for the average ten-to-twelve year old.

3. They should be problems that require the child to predict various possibilities, to verify and to search for alternatives;

to combine two or more rules or principles; to consider more than one variable in a situation; in short, to engage in "open search" behavior, considering various conditions and synthesizing the relevant data to reach a satisfactory solution.

4. While they should not be so easy to solve that they do not elicit search behavior, they should also not be so difficult or confusing that the majority of children in grade six would refuse to attempt them.

These criteria are essentially similar to those cited by Kilpatrick (1967) for his investigation into the problem solving behavior of eighth-grade students. A discussion of each of the problems which were chosen for the present study will indicate how each one fits the criteria. Problem I was taken from the Larson study; the others were chosen from various sources such as those sometimes used for "enrichment" or "recreations" in the elementary school.

Problem I - "the lost ball".

> Just for practice let's begin with a game called "Twenty Questions." Here is a chart of a field. Let's say that a boy lost his baseball somewhere in this field. I know where it is, but you do not. You have twenty questions to see if you can find out where it is (Larson, 1966, p. 153).

The "chart" in this problem is a sheet of manilla tag which is divided into one-inch squares, twelve by eleven inches, or one hundred and thirty-two in all. If the child structures his search for the ball in a systematic manner, he could locate the "missing" ball in seven questions by the elimination of alternatives. The problem was chosen for the present study because the game-like setting would relieve any test anxiety experienced by the subject,, it was a problem all the

children could "solve" since the experimenter can make sure that the ball is "found" before the twenty questions have expired, and finally, it would provide an opportunity to establish a desirable rapport between the interviewer and the subject.

Problem II - "how many socks"

> Jane was getting dressed for a school concert when a thunderstorm caused all the lights in the house to go out before she had picked out a pair of matching socks from her drawer. Her father is waiting in the car to drive her to the school. They have just enough time to get there before the concert starts. She knows that she has only yellow, white and blue socks in her drawer and that any of these colors will match the dress she is wearing. How many socks should Jane take with her so that she can be sure of having two socks the same color to put on in the car? (Explain that the socks are not arranged in pairs in the drawer.)

This problem is characteristic of several to be found in arithmetic "puzzle" books. It calls for the subject to predict that if Jane takes three socks each one could be a different color. A further hypothesis must then be made: "If she takes one more sock, then it has to be the same as one of the other three because there are only three different colors (validation)." Another alternative could also be considered: "If she doesn't pick up three different colors with her first three choices, then she has to have two that are the same color before she has even picked up the fourth sock."

Problem III - "how much copper"

> A nickel (five cents) is made of both nickel and copper. For every pound of nickel that goes into the mixture, there are three pounds of copper. How many pounds of copper would be needed to make 24 pounds of the nickel coins?

It was expected that most children would attempt to set up a rate equation using 1 per 3 equal to some unknown per 24. When this equation failed to produce a verifiable solution, the subject would

then have to search for a new variable, the quantity 4. A successful solution requires the subject to restructure the problem so that the proportion of three pounds of copper per four pounds of the mixture is arrived at. Would the search be systematic, or would the subject engage in trial and error, guessing and the verifying each guess?

Problem IV - "chess and checkers"

> All of the students in a certain class know how to play either chess or checkers, but some know how to play both games. There are 14 students who know how to play chess; there are 16 students who can play checkers; but there are 10 students who can play both chess and checkers. Can you figure out how many students there are in the class?

For a subject to accept this situation as a real problem, he must first of all grasp the idea that the ten students who play both games are also included in the chess players and in the checker players. Otherwise, no problem exists and the child would simply add up 14, 16 and 10. Once the class inclusion aspect has been sensed, however, the subject must still search for a solution. Several approaches or hypotheses are possible: remove the ten who play both games from the the fourteen chess players and the sixteen checker players, and then add up the three disjoint groups; add the fourteen and the sixteen, but then the ten who play both must be subtracted from the total; or count half of the ten, who play both games, as chess players, and the other half as checker players, subtracting the remainder, five, from both the fourteen and the sixteen. It was expected that some subjects would attempt to verify their solutions by drawing some kind of diagram to represent the situation.

<u>Problem V</u> - "<u>the vinegar problem</u>"

> A supermarket bought 275 gallons of vinegar in large barrels which had to be emptied right away and sent back to the factory. The manager of the supermarket found that he had only 168 empty gallon jars that he could fill with vinegar. He knew that these would not be enough to hold all the vinegar, so he decides to put the rest of the vinegar into pint containers. Could you figure out how many pints he will need?

It was not the intention of the investigator to test the computational skills of the children, but only to test the flexibility and appropriateness of the strategies they would use to solve the problems. The quantities chosen for Problem V were fairly simple to convert to other units and there would be no remainders when it was necessary to divide. The subjects were told that if they needed help with the computations in any of the problems, they had only to ask and the interviewer would assist them. They could ask questions like, "Are there four quarts in a gallon?" and receive a yes or no answer. In this way, problems involving a fair amount of computation should not have been unduly difficult, providing that the child had at his disposal some kind of strategy for seeking a solution.

One of the reasons for including Problem V was that it involved solving several sub-problems: changing the quarts to gallons; subtracting both the 24 gallons and the 168 gallons from the original quantity; and finally, converting the remaining gallons of vinegar into pints. These sub-problems necessitated the maintenance of search behavior over a longer period of time than did the other problems, as well as a certain amount of systematization (remembering and using solutions from the sub-problems). It was, however, not as easy to validate the final solution as was the case with the other problems in the test.

Problem VI - "father's age"

> A man married at the age of 25. His wife died 15 years after their marriage, leaving a daughter who was only 11 years old. After 9 more years, the daughter married a man who was 4 years older than she was. When her father died, the daughter's husband was 45 years old. Is there any way that you could figure out how old her father was when he died?

In order to accept this as a real problem, the subject would have to grasp the relationship between the father's age and the son-in-law's. This relationship must be analyzed in terms of the life lines of the three people involved, and requires a subject to search for some way of comparing their respective ages at some point in time. It was anticipated that this problem would elicit the formulation of hypotheses and the verification of outcomes as the subject tried to find the father's age. How systematic this search would be might reveal the efficiency and flexibility of the subject's strategies.

Problem VII - "gift of money"

> Two brothers, Floyd and David, receive a gift of money each Christmas to be divided equally between the two boys. The total amount of money that their uncle sends for both of them is always equal to the product of their ages (explain the meaning of product). This past Christmas, in 1969, the uncle sent $36. The Christmas before last, in 1968, he sent $22 for the boys to divide between them. Could you figure out how old the boys were last Christmas; that is, what were their ages in 1969 when the uncle sent them $36?

As in the case of Problems IV and VI, for the subject to engage in the process of problem solving at all, he had to grasp certain relationships. In Problem VII it was the relationship between the amount of money which the uncle sent them in 1969 and the amount which the boys received the previous Christmas. In other words, the subject must grasp the idea that, when Floyd and David received twenty-two dollars; that is, in 1968, the two boys would each have

been one year younger. A systematic strategy would involve considering all the possible combinations of ages for the two boys when the product of their ages was thirty-six and when it was twenty-two. If the subject were to start with the smaller number, he would soon find that there are only two possibilities for the year 1968, and it would then be fairly easy to test the validity of both combinations in terms of the amount of money they received the following year. A specific or non-systematic strategy would involve random guessing with the hope that the right combination might turn up. This problem seemed like a particularly good one for analyzing the strategies used by the different subjects.

II. PILOT STUDY

A pilot study was carried out in February, 1970, using a sample of ten grade six students from a school not included in the study. The purpose of the pilot study was two-fold: to determine the appropriateness of the items chosen for the Strategies Test and to refine the interviewing techniques to be followed in the research. The following conclusions were drawn as a result of the pilot study:

1. The problems chosen for the test were appropriate. Most of the children interviewed were quite willing to attempt all of the items without showing undue fatigue. All the children appeared to enjoy the game of "Twenty Questions."

2. The overt responses of the subjects as they "thought aloud" seemed to be indicative of their patterns of thought (strategies).

3. It would be necessary to follow a carefully standardized procedure in administering the test so that the interviewer did not

provide cues to some subjects and not to others by means of questions and remarks made during the problem-solving interviews.

4. Some means should be found to alleviate the frustration experienced by some of the subjects as they became aware that they had not found a verifiable solution, but had exhausted all the alternatives open to them.

III. ADMINISTRATION OF THE TEST

The Strategies of Problem Solving Test was administered on an individual basis to the eighty grade six students in the sample. All interviews were conducted by the investigator. Since comparisons were to be made between the subjects in the Inquiry group and those in the Non-Inquiry group, it was important that the physical setting be carefully controlled and that a standardized interview procedure be maintained throughout. On the other hand, since the interviews were intended to be inquiry sessions, it was important that a good rapport be established and test-anxiety be alleviated.

Suchman's (1966) recommendation that the teacher should respond positively to the student and neutrally to the product of the child's thinking (p. 17) was taken as a guide in structuring the interviews. The subject was encouraged to think aloud and to ask questions as he worked at the problems. Questions asked for purposes of clarification by the subject were answered by referring back to the information given in the problem. Computational assistance was given when requested but the interviewer refrained from structuring the problem or indicating which operations should be used to solve it. Every attempt was made to have the subject talk about the problem and to explain how he had reached a certain conclusion: e.g. "Do you think you have reached a

correct solution?"; "Why do you think that's the right answer?" or "Why aren't you sure?"

As a result of the pilot study a list of "permissible" questions and comments had been drawn up for each problem on the test. This list (Appendix C) was carefully adhered to by the investigator.

The interviews were held in a conference-type room in the schools during the school day. When a subject arrived, he was greeted by name and a few moments were spent talking about the topic which the subject had indicated in his written composition as being of particular interest to him: e.g. astronomy, horses, pets, etc. Care was taken to ensure that the subject was not unduly disturbed by the presence of the tape recorder which lay on the table in full view throughout the interview.

Reasons for the interview were explained as follows:

> I think you know (name of subject), that I am working on a project to find out what kinds of problems are interesting for boys and girls in grade six. This is how you can help me with the project. I've chosen some problems that I think are interesting. I'm going to ask you to try to figure them out so that I can see if you find them interesting too.
>
> In order to give you a better chance to figure out the problems, you may ask me any questions that you need to. When you ask a question that I cannot or will not answer, I shall tell you. Don't worry about asking the wrong questions. There really are no right or wrong questions.
>
> Some of the problems may involve doing some arithmetic computation, so I have provided a pencil and paper here for you to use if you need to. If you want me to check the accuracy of your adding, dividing, etc., I'll be glad to do so. But I won't offer to help unless you ask me to. Remember to talk as much as you like as you think through the problems, and to ask any question at all that you think might help you to figure them out. Are you ready?
>
> All right. Just for practice, let's begin with a game called "Twenty Questions." Have you ever played Twenty Questions?

Care was taken at each interview to ensure that the subject understood the problem situation and wanted to become involved in

finding a solution. After each problem had been read aloud, as animatedly as possible, a card on which the problem had been printed in large type was placed in front of the subject for ready reference and re-reading if necessary. No indication was given as to the validity of the solutions reached by a subject. Questions such as, "Is that right?" were countered with "What do you think? Do you think it's right?"

Every attempt was made to encourage a subject to continue thinking about a problem until some kind of a solution had been reached. If he stated that he didn't think he could find the answer, he was asked, "Do you think there is enough information given for you to figure out this problem?" or, "What more would you need to know so that you could figure out the answer?" This permitted a subject to cease working on a problem without seeming to have failed, or without becoming overly-anxious about lack of success. It was felt that a feeling of failure experienced early in the interview could have had implications for success or failure on later problems and therefore should be eliminated as much as possible.

Most of the interviews lasted approximately forty-five minutes; however, no time limit was set. The subjects were allowed to continue working at a problem as long as they wished to do so. At the end of the interview the subject was thanked for his participation in the research project. To ensure against contamination of the sample, he was asked not to discuss the problems or the game of "Twenty Questions" with his classmates. There was no reason

to believe that contamination did in fact occur in any of the schools included in the study.

IV. SCORING OF THE TESTS

The criteria set out by Larson to score Problem I (the lost ball) were used in this study. These are given in detail in Appendix C and will not be included here since it was decided to use the problem for practice only and not to include it in the total test score in the statistical analysis.

It had been anticipated that strategies similar to those identified by Larson for her "lost ball" problem would appear in the protocols used by the subjects to solve the mathematical problems as well. However, the problem solutions did not lend themselves to Larson's categories. There were nonetheless obvious differences in the way that students oriented themselves to the problems, organized their search for a solution and validated their answers. A method of scoring based on these categories was therefore set up.

For ease and objectivity of scoring, each problem solution was considered in three ways: (1) the extent to which a subject sensed or structured the problem before attempting to find a solution; (2) predicted or hypothesized one or more avenues through which a solution could be reached; (3) verified or validated a suggested solution. For each of these processes - sensing, predicting and verifying - a value of two was assigned with a maximum score of six for each problem. The scores were given in accordance with specified criteria for each problem (Appendix C) and the system proved to be quite workable.

Since the validity of the findings would be dependent

upon the reliability of the scoring, it was decided to establish a measure of reliability for the scoring of the six problems. Three judges, two capable graduate students in the field of mathematics education and the experimenter, were given the transcribed protocols of ten randomly chosen subjects and were required to score these independently. A comparison of the values assigned by the three judges for each of the problems is given in Table IV.

A coefficient of interscorer reliability was calculated using Winer's formula for Single Factor Experiments with Repeated Measures (1962, pp. 105-132). The adjusted reliabilities were .98, .96, .98, .99, .98 and .99 for Problems II to VII respectively. The overall reliability coefficient for the test was .99.

V. SUMMARY

This chapter contained a discussion of the construction and administration of the Strategies of Problem Solving Test. A rationale was established for the choice of items included in the test. The first item, Larson's "lost ball" was used as a practice problem; the remaining six problems were of a quantitative nature and required subjects to engage in a sensing-predicting-verifying cycle of mental activity while searching for a solution. The tape-recorded protocols of the subjects as they "thought aloud" were analyzed and scored according to a set of established criteria.

TABLE IV

COMPARISON OF TEST SCORES FOR TEN SUBJECTS WITH THREE JUDGES

Subject Number	Problem:	II			III			IV			V			VI			VII			Test Total		
	Judge:	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
101		6	6	6	6	6	6	3	4	3	6	6	6	6	6	6	6	6	6	33	34	33
102		5	5	6	2	2	1	2	1	1	2	3	3	1	2	1	6	6	6	18	19	18
201		6	6	6	5	6	6	3	3	2	6	6	6	6	5	6	6	6	6	32	32	32
202		6	5	5	4	5	4	6	6	6	6	6	6	6	6	6	6	6	6	34	34	33
301		1	1	1	2	1	1	3	3	2	0	0	0	1	3	2	4	4	4	11	12	10
402		2	2	1	3	1	2	1	1	1	2	3	3	0	0	0	0	0	1	8	7	8
503		3	2	2	2	1	1	1	2	1	1	2	2	4	3	4	1	0	1	12	10	11
604		5	6	6	5	4	4	6	6	6	6	6	6	3	3	3	0	0	1	25	25	26
705		3	1	2	3	3	4	5	4	4	6	6	6	1	2	1	1	0	2	19	16	18
806		1	1	1	3	2	2	1	1	1	3	4	3	6	6	6	3	3	3	17	17	16

CHAPTER FIVE

RESULTS OF THE INVESTIGATION

This chapter reports an analysis of the differences between the Inquiry and Non-Inquiry groups on several variables as well as the results of testing the hypotheses. Other findings are also presented.

I. DIFFERENCES BETWEEN GROUPS

It had been assumed that the two major groups in the sample were roughly representative of the same population and that any differences found between them in problem solving ability might be attributable to the different kinds of arithmetic programs which they had experienced in the sixth grade. In order to determine whether they were indeed relatively equivalent, the two groups were compared on the variables of age, intelligence, language achievement and problem solving achievement. This analysis of variance is presented in Table V.

The only variables on which the differences in means between the two groups approached significance was on the Iowa Problem Solving subtest. This indicated that the two groups were somewhat different in achievement on a conventional arithmetic problem test and that steps should be taken to control for this variance when testing Hypothesis 1.

Although the mean intelligence, both verbal and non-verbal was almost the same for both groups, an examination of the distribution of intelligence within the groups showed that the range of I.Q. was

greater for the Inquiry group. Similarly the distribution of age for the two groups showed that the subjects in the Inquiry group were somewhat younger than those in the Non-Inquiry group. On the basis of this variance in distribution of intelligence and age (Tables VI and VII), it was decided to control for these variables as well as for problem solving achievement.

TABLE V

ANALYSIS OF VARIANCE BETWEEN INQUIRY AND NON-INQUIRY GROUPS

Variables	Inquiry		Non-Inquiry			
	Mean	S.D.	Mean	S.D.	F	p
Age (months)	140.63	6.78	141.85	5.89	.74	.39
Verbal I.Q.	107.98	16.81	107.95	12.42	.00	$>$.99
Non-Verbal I.Q.	109.10	14.31	109.40	10.14	.01	.91
Language (5-point scale)	2.80	1.13	3.20	1.20	2.34	.13
Iowa Problem Test[a]	16.00	5.03	18.13	4.98	3.60	.06

[a]Maximum score - 31

II. RESULTS OF TESTING THE HYPOTHESES

Hypothesis 1.

There is no significant difference in problem solving ability, as measured by the Strategies of Problem Solving Test, for students in the Inquiry group and those in the Non-Inquiry group, when controlling for age, intelligence and problem solving achievement.

TABLE VI

DISTRIBUTION OF I.Q. FOR STUDENTS IN THE SAMPLE
N = 80

Interval	Inquiry Group Verbal	Inquiry Group Non-Verbal	Non-Inquiry Group Verbal	Non-Inquiry Group Non-Verbal
Below 80	2	1		
80-89	5	1		2
90-99	7	10	11	4
100-109	6	9	13	15
110-119	7	9	9	14
120-129	10	7	5	14
130-139	2	1	1	4
140-149	1	2	1	1
Total	40	40	40	40

TABLE VII

DISTRIBUTION OF AGE FOR STUDENTS IN THE SAMPLE
N = 80

Interval in Years and Months	Inquiry	Non-Inquiry
10-6 to 10-11	5	3
11-0 to 11-5	7	3
11-6 to 11-11	14	20
12-0 to 12-5	10	11
12-6 to 12-11	3	3
13-0 to 13-5	1	

Appropriate models were constructed so that this hypothesis could be tested using Multiple Linear Regression Analysis. As shown in Table VIII, the difference in means between the two groups was found to be highly significant when age, I.Q. and problem solving achievement were covaried. Even when no controls were applied the difference was found to be significant at the .02 level of probability. It therefore appeared that the hypothesis should be rejected.

Supplement to Hypothesis 1. Since it had been established that there was a significant difference in problem solving ability in favor of the Inquiry group, it seemed desirable to investigate the source of this difference. Were each of the classroom subgroups contributing equally to this difference?

An examination of the means for each of the subgroups, shown in Table IX, revealed that one classroom subgroup (Nc) seemed to be contributing more to the difference between the two major groups than any of the other three Non-Inquiry subgroups. Furthermore, another Non-Inquiry subgroup (Nb) had a mean that was actually higher than the means of two of the Inquiry subgroups (Ic and Id), and also higher than the mean for the total Inquiry group.

In order to establish statistically the significance, if any, of the differences among the subgroups, their means were submitted to a Scheffé test. It was not deemed necessary to use adjusted means for this test, since the difference between the two major groups had been shown to be significant even when no controls were applied to partial out the variance due to age, intelligence and problem solving achievement. The probability matrix for the Scheffé Multiple Comparison of Means,

TABLE VIII

ANALYSIS OF VARIANCE ON STRATEGIES OF PROBLEM SOLVING TEST:

INQUIRY NON-INQUIRY GROUPS (Hypothesis 1)

Mean Score		Degrees of freedom		F-Ratio	p
Inquiry	Non-Inquiry	num.	den.		
20.13	15.90	1	75	11.7	.001

TABLE IX

A COMPARISON OF MEANS AND STANDARD DEVIATIONS FOR THE STRATEGIES

OF PROBLEM SOLVING TEST[a]: SUBGROUPS (N=10)

	Inquiry			Non-Inquiry	
Class Code	Mean	S.D.	Class Code	Mean	S.D.
Ia	23.10	6.47	Na	16.80	8.29
Ib	21.20	8.43	Nb	20.20	6.01
Ic	16.20	7.70	Nc	10.10	6.76
Id	20.00	8.03	Nd	16.50	8.02
Total Inquiry	20.13	7.82	Non-Inquiry	15.90	7.95

[a]Maximum score = 36

presented in Table X, shows that there was a significant difference in means only between Inquiry subgroup Ia and Non-Inquiry subgroup Nc, when each of the subgroups is compared with every other subgroup. This analysis was sufficiently revealing so as to suggest considerable caution in interpreting the findings related to Hypothesis 1. It is evident that the mean score of only _one_ Inquiry subgroup was significantly higher than the mean of only _one_ of the Non-Inquiry subgroups. Furthermore, _three_ of the Non-Inquiry subgroups actually scored means which were higher than _one_ of the Inquiry subgroups, although the differences were not significant. Results of testing Hypothesis 1. are therefore somewhat equivocal and will be discussed further in Chapter Six.

Hypothesis 2.

There is no significant difference between male and female subjects in problem solving ability as measured by the _Strategies of Problem Solving Test_.

Multiple Linear Regression Analysis was used to test this hypothesis. The difference in means for the male and female subjects was found to be not significant at the .05 level when an F-ratio was calculated (Table XI). Hence the hypothesis was not rejected. Males and females do not differ significantly in problem solving ability as measured by the _Strategies of Problem Solving Test_.

Hypothesis 3.

There is no significant relationship between a subject's problem solving ability as measured by the _Strategies of Problem Solving Test_ and:

(a) Language Achievement

TABLE X

PROBABILITY MATRIX FOR SCHEFFE MULTIPLE COMPARISON OF MEANS ON STRATEGIES OF PROBLEM SOLVING TEST: SUBGROUPS (n=10)

Subgroup	Ia	Ib	Ic	Id	Na	Nb	Nc	Nd
Ia	--	.99	.75	.99	.82	.99	.05	.79
Ib		--	.94	>.99	.97	>.99	.16	.96
Ic			--	.99	>.99	.98	.85	>.99
Id				--	.99	>.99	.29	.99
Na					--	.99	.78	>.99
Nb						--	.27	.99
Nc							--	.82
Nd								--

TABLE XI

ANALYSIS OF VARIANCE ON STRATEGIES OF PROBLEM SOLVING TEST: MALES AND FEMALES (Hypothesis 2)

Mean Score		Degrees of freedom		F-Ratio	p
Males	Females	num.	den.		
23.20	20.83	1	78	1.57	.21

(b) Verbal Intelligence

(c) Non-Verbal Intelligence

(d) Age

(e) Arithmetic Problem Solving Achievement.

(a) Problem Solving Ability and Language Achievement. Table XII gives the correlations of language achievement with each of the problem subscores and with the total score on the Strategies Test. Problem III and Problem VI showed a statistically significant, but low, correlation with language achievement. The correlation of language achievement with the total score on the Strategies Test was not significant at the .05 level and therefore, the hypothesis was not rejected. There is no significant relationship between language achievement and problem solving ability as measured by the Strategies of Problem Solving Test.

(b) Problem Solving Ability and Verbal Intelligence. The correlation coefficients and their probabilities for verbal intelligence and the Strategies Test subscores and total are given in Table XIII. Correlations significant at the .01 level appear only for Problems III, V and VI. There is also a correlation of .35 for the total test score which was significant beyond the .01 level and therefore, Hypothesis 3 (b) was rejected. There is a significant relationship between problem solving ability and verbal intelligence.

(c) Problem Solving Ability and Non-Verbal Intelligence. Table XIV gives the correlation coefficients and probabilities for non-verbal intelligence and the Strategies Test subscores and total. Correlations significant at the .01 level appear for Problems II, III, V, VI and VII, with III having the highest coefficient. Since there

TABLE XII

CORRELATIONS: STRATEGIES TEST SUBSCORES AND TOTAL - LANGUAGE ACHIEVEMENT

(Hypothesis 3 a)

Problem	I	II	III	IV	V	VI	VII	Total
r	.18	.04	.26	-.15	.15	.25	.10	.19
p	.25	.67	.02	.19	.18	.02	.37	.08

TABLE XIII

CORRELATIONS: STRATEGIES TEST SUBSCORES AND TOTAL - VERBAL INTELLIGENCE

(Hypothesis 3 b)

Problem	I	II	III	IV	V	VI	VII	Total
r	.14	.20	.31	-.04	.34	.34	.12	.35
p	.20	.08	< .01	.72	< .01	< .01	.28	< .01

TABLE XIV

CORRELATIONS:STRATEGIES TEST SUBSCORES AND TOTAL-NON-VERBAL INTELLIGENCE

(Hypothesis 3 c)

Problem	I	II	III	IV	V	VI	VII	Total
r	.18	.28	.41	-.04	.32	.49	.29	.48
p	.12	.01	< .01	.71	< .01	< .01	.01	< .01

was a correlation of .48 for the total test which was significant beyond the .01 level, Hypothesis 3 (c) was rejected. There is a significant relationship between problem solving ability and non-verbal intelligence.

(d) Problem Solving Ability and Age. Table XV gives the correlation coefficients and probabilities for age and the Strategies Test subscores and total. It was noted that, while Problems II and V, as well as the total score, do correlate significantly with age, all of the coefficients are negative. This result is not too surprising considering the small range of ages within the sample (about two and a half years), and considering also that ten subjects (one-eighth of the sample), whose ages were eleven years and two months or less, were all on a five-year program and were all rated as above average in intelligence. The five-year program in the Edmonton Public School System selects students of superior ability and accelerates their rate of progress in the elementary school by one year over a five year period. It should also be noted that age correlated negatively with every other variable in the study as can be seen by examining the intercorrelation matrix for the major variables (Appendix D).

With the reservations noted above, it must be concluded that, in this study, there was a significant, but negative, correlation between problem solving ability and age. Hypothesis 3 (d) was rejected.

(e) Problem Solving Ability and Arithmetic Problem Solving Achievement. Hypothesis 3 (e) was intended to test the relationship between ability as measured by the Strategies Test and achievement as measured by the Iowa Problem Solving Subtest. Table XVI presents the correlations and probabilities for these two instruments. Problems III, V, VI and VII correlated significantly with the Iowa test. There

was a correlation coefficient of .40, significant beyond the .01 level and the hypothesis was therefore rejected. There is a significant relationship between problem solving ability and arithmetic problem solving achievement.

Summary of Hypothesis 3. Part (a) was not rejected . Problem solving ability did not correlate significantly with language achievement. Parts (b), (c), (d) and (e) were rejected. With certain limitations for age, problem solving ability correlates significantly with both verbal and non-verbal intelligence, with age, and with arithmetic problem solving achievement.

Hypothesis 4.

There is no significant difference between subjects classified as tentative and those classified as non-tentative in:

(a) Language Achievement

(b) Problem Solving Ability.

Tentativeness and Language Achievement. Using appropriate models, Hypothesis 4 (a) was tested by Multiple Linear Regression Analysis. The results are presented in Table XVII. Since subjects classified as tentative were not rated significantly higher in language achievement, as measured in this study, Hypothesis 4 (a) was not rejected. There is no significant difference in Language Achievement for subjects who exhibit tentativeness during problem solving and those who did not exhibit tentativeness.

Tentativeness and Problem Solving Ability. Hypothesis 4 (b) was also tested by Multiple Linear Regression Analysis. Findings presented in Table XVIII show that there is a highly significant

TABLE XV

CORRELATIONS: STRATEGIES TEST SUBSCORES AND TOTAL - AGE (Hypothesis 3 d)

Problem	I	II	III	IV	V	VI	VII	Total
r	-.12	-.32	-.17	-.08	-.23	-.14	-.06	-.27
p	.28	$<$.01	.14	.50	.04	.20	.59	.02

TABLE XVI

CORRELATIONS: STRATEGIES TEST SUBSCORES AND TOTAL - IOWA TEST (Hypothesis 3 e)

Problem	I	II	III	IV	V	VI	VII	Total
r	.12	.15	.27	.09	.39	.40	.27	.40
p	.31	.19	.02	.44	$<$.01	$<$.01	.01	$<$.01

TABLE XVII

ANALYSIS OF VARIANCE IN LANGUAGE ACHIEVEMENT FOR TENTATIVE AND NON-TENTATIVE SUBJECTS (Hypothesis 4 a)

Mean Score (max. 5)		Degrees of freedom		F-Ratio	p
Tentative	Non-Tentative	num.	den.		
Problem III					
3.3	2.8	1	78	2.80	.10
Problem VII					
3.2	2.8	1	78	2.41	.12

difference in means on the <u>Strategies of Problem Solving Test</u> between subjects classified as tentative and those classified as non-tentative, even when the score for the problem on which the classification was based is omitted from the test total. This was done so as to avoid prejudicing the findings since subjects who scored high on Problem III or Problem VII were usually classified as being tentative for those two problems respectively.

Hypothesis 4 (b) was rejected. There is considerable evidence that tentativeness is closely associated with successful strategies of problem solving. Subjects classified as tentative scored significantly higher than did non-tentative subjects even on problems other than those on which the classification of tentativeness was based.

III. OTHER FINDINGS

The tape-recorded protocols of the problem-solving interviews represented a wealth of data which lend themselves to descriptive rather than statistical analysis. Some of the findings will be given here.

<u>Tentativeness</u>

<u>Problem III</u>. Twenty-four subjects were identified as having used supposition and/or conditionals during their search for a solution to Problem III. Of these, only eight, or thirty-three per cent, were rated above average in language achievement. There was little evidence therefore to support the hypothesis, expressed by Loban (1963), that high achivers in language use more tentative expressions than do low achievers.

Seventeen of the twenty-four subjects who were classified as

tentative obtained the maximum score of six for Problem III. In other words, tentativeness was associated with successful problem solving seventy-one per cent of the time. It was also found that, of the eighteen subjects who had received the maximum score for Problem III, only one subject was not represented in the tentatives. Successful problem solving for Problem III was therefore associated with tentativeness ninety-four per cent of the time.

An examination of the total scores for the Strategies Test yielded further evidence that successful problem solvers exhibited a greater degree of tentativeness than did those who were less successful on the Strategies Test. Of the fourteen subjects who attained the highest total score for the total test: i.e. a score of twenty-six or higher, every one was also a member of the "tentatives" group for Problem III. (Thirteen of these were later shown to be tentative for Problem VII as well.) The data on which the findings with respect to Problem III were based is presented in Table IXX.

Problem VII. Thirty-three subjects were classified as tentative on the basis of comments made during the solution of Problem VII. Seventeen of these had also been judged tentative for Problem III. Of the thirty-three tentatives, twenty-six obtained a maximum score of six for Problem VII. In other words, eighty per cent of those who exhibited tentativeness for the problem had also arrived at a correct solution. There was no subject who was assigned the maximum score and who was not classified as tentative for Problem VII. Table XX presents the data on which these conclusions are based.

TABLE XVIII

ANALYSIS OF VARIANCE ON STRATEGIES OF PROBLEM SOLVING TEST FOR TENTATIVE AND NON-TENTATIVE SUBJECTS (Hypothesis 4 b)

Mean Score (max.=30)[a]		Degrees of freedom		F-Ratio	p
Tentative	Non-Tentative	num.	den.		
Problem III					
19.75	13.43	1	78	16.49	$< .001$
Problem VII					
18.76	12.45	1	78	21.67	$< .001$

[a]Total test score omits subscore for Problems III and VII respectively.

TABLE IXX

PERFORMANCE OF SUBJECTS CLASSIFIED AS TENTATIVE FOR PROBLEM III

N = 24

Inquiry Group				Non-Inquiry Group			
Student Number	Language Rating	Sub-score III	Total	Student Number	Language Rating	Sub-Score III	Total
101	4	6	33	501	4	6	28
106	3	6	32	502	5	6	28
110	4	6	30	506	2	6	25
201	3	6	32	510	5	3	7
202	3	4	33	601	3	3	27
204	5	6	18	602	4	6	29
205	3	6	33	604	3	4	26
206	4	3	18	605	2	3	13
207	2	2	12	608	3	6	18
305	3	6	21	801	5	6	32
401	3	6	33	807	2	6	15
404	3	6	32				
408	2	6	23				

TABLE XX

PERFORMANCE OF SUBJECTS CLASSIFIED AS TENTATIVE FOR PROBLEM VII

N=33

Inquiry Group				Non-Inquiry Group			
Student Number	Language Rating	Sub-score VII	Test Total	Student Number	Language Rating	Sub-score VII	Test Total
101	4	6	33	501	4	2	28
102	2	6	18	502	5	6	28
103	2	4	17	504	3	3	17
105	2	4	25	506	2	6	25
106	3	6	32	601	3	6	27
109	1	6	17	602	4	6	29
110	4	6	30	609	5	6	24
201	3	6	32	701	4	6	17
202	3	6	33	706	3	6	23
204	5	2	18	801	5	6	32
205	3	6	33	806	3	3	16
206	4	6	18	810	4	6	19
209	1	6	14				
210	5	4	20				
304	1	6	15				
305	3	6	21				
306	2	6	25				
307	5	6	24				
401	3	6	33				
404	3	6	32				
406	3	6	22				

Trial-and-Error

Another analysis which proved particularly intersting involved the classification of the protocols on the basis of the procedures used by the subjects to solve one or more of the problems. These were categorized as Equation (algorithmic solution), Deduction (logical reasoning) and Trial-and-Error (including successive approximations), or any combination of the three procedures. This categorization was also used by Kilpatrick (1967) in his study of the procedures used by eighth-grade pupils as they attempted to solve a battery of mathematical problems.

In the present investigation, the use of either trial-and-error or successive approximation was associated most of the time with both tentativeness and with a correct solution for Problem III. Deduction or equation, on the other hand, rarely yielded a correct solution. The trial-and-error method was also characterized by frequent use of conditionals and supposition: therefore the relationship with tentative thinking seems to be a natural one. Of the nineteen students who used trial-and-error, either right away or after another approach had failed to produce a verifiable answer, all were also classified as being tentative for Problem III.

These findings will be discussed further in Chapter Six.

IV. SUMMARY

This chapter contained an analysis of the differences observed between the Inquiry and Non-Inquiry groups on several variables: age, intelligence, language achievement and arithmetic problem solving achievement. It also reported the results of testing the four null hypotheses which were associated with the purposes of the study as outlined in Chapter One.

The first purpose of the study was to determine whether an inquiry approach to mathematics teaching would have any significant effect on the problem solving ability of grade six pupils as measured by the Strategies of Problem Solving Test. It was found that subjects who had been taught by an inquiry approach did score significantly higher than those who were taught by a non-inquiry approach. An analysis of the differences in means for the subgroups (ten subjects from each of the eight participating classrooms) revealed that there was a significant difference in means only between one Inquiry subgroup and one Non-Inquiry subgroup.

No sex differences were observed to be statistically significant when ability to solve problems was measured by the Strategies Test. Age, verbal and non-verbal intelligence, and arithmetic problem solving achievement were found to be significantly related to problem solving ability. Language achievement, as measured in the study, was not found to be significantly related to either problem solving ability or to tentativeness. Tentativeness and the ability to use trial-and-error as a method of solving arithmetic problems were found to be associated with superior problem solving ability. Implications from these findings will be discussed in the final chapter.

CHAPTER SIX

SUMMARY, CONCLUSIONS, IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

I. SUMMARY OF THE INVESTIGATION

The purpose of this study was: (1) to investigate the effectiveness of an inquiry approach to the teaching of arithmetic in grade six for the development of efficient strategies of problem solving; (2) to examine the relationships among language achievement, tentative thinking and problem solving.

The randomly chosen sample consisted of eighty grade six students, ten from each of eight classrooms located in seven schools in the city of Edmonton, Alberta. Forty of these students were from classrooms which had been identified as using an 'inquiry' approach and forty from classrooms using a 'non-inquiry' approach to the teaching of arithmetic.

A Strategies of Problem Solving Test, consisting of seven verbal problems, was administered by means of individual interviews during which subjects were asked to "think aloud", to explain why they chose a particular approach to a problem, and to give reasons for believing that they had arrived at a correct or incorrect solution. The tape-recorded protocols were transcribed and scores assigned to each subject on the basis of the strategies which were used to solve the problems. Mean scores for the Inquiry group were compared to mean scores for the Non-Inquiry group and tested statistically to determine the significance of the difference between them.

The two groups were also compared on the variables of age,

intelligence and language achievement, as well as on problem solving achievement measured by a written standardized test of arithmetic problem solving. The subjects' protocols for two of the problems from the Strategies Test were examined for evidence of tentative thinking, and a tentative or non-tentative rating was assigned. A statistical analysis of the relationship between these variables and problem solving ability (measured by the Strategies Test) was carried out.

The conclusions based on the findings, implications for education and suggestions for further research are presented in this chapter.

II. CONCLUSIONS

The first question which was posed at the beginning of this study was: "Are there identifiable strategies which children adopt as they engage in a search for a problem solution?" The results seem to indicate that such strategies do exist and that they are related to a cycle of mental activity during which a child first of all <u>senses</u> or grasps the significance of a problem, seeks different ways of <u>predicting</u> or hypothesizing while searching for a solution, and attempts to <u>verify</u> his predictions or to settle conjectures. If validation of a solution is not possible, he may often start over again, sensing, predicting and verifying. The extent to which an individual is able to engage in these strategies will usually determine his success or failure in finding an appropriate solution. The conclusion is reached that developing flexible strategies which permit children to search for relationships and patterns as well as for solutions to problems, may indeed be one of the important goals of mathematics education.

Some evidence has been found to support the thesis that using an inquiry approach to the teaching of arithmetic enhances the problem solving strategies of grade six students. However, there is a strong possibility that a gain in one area of mathematical competence may be achieved at the expense of another. The Inquiry group Ia which scored the highest on the Strategies Test was also the same one which scored the lowest of all eight subgroups (Inquiry and Non-Inquiry) on a conventional standardized arithmetic problem test (Appendix D). It is not known whether the low performance of this particular subgroup was due to computational errors, errors in understanding the problem, or simply to a lack of practice in writing this kind of a test, since the design of the study did not include these variables. It is important, however, to note that the subgroup Ia came from the classroom which had completely converted to an activity or investigations approach. The other three Inquiry classrooms, particularly Ic and Id, had retained to some degree a textbook approach, and were using a modified form of the inquiry method method, based on individual, rather than classroom, inquiry. (A description of the programs in all of the classes was given in Chapter III.)

It is interesting to note that Non-Inquiry subgroup Nb not only had the highest mean of all the subgroups on the Iowa test, but also the third highest mean on the Strategies of Problem Solving Test. This particular room could also be described as using a blend of the traditional approach to teaching arithmetic, with an emphasis on developing computational skills, and what could be termed a more modern, "teaching for understanding," approach. Perhaps it was

combining the best of both methodologies.

The relationship of problem solving to language achievement, as it was measured in this study, was shown not to be significant, although both language and the Strategies Test correlated significantly with verbal and non-verbal intelligence (Appendix D). Tentativeness, however was shown to be closely related to successful strategies of problem solving. Subjects who engaged in tentative thinking during the solving of a problem were more likely to reach a correct solution than those who did not seem capable of considering more than one alternative.

Contrary to Loban's hypothesis (1963), subjects who had a high language rating did not appear to engage in more tentative thinking than did those with a low language rating. This is partially explained by the fact that the language sample for which the rating was given was a vastly different situation than the one on which the tentativeness rating was assigned. The former was a personal written composition; the latter was based on oral language used in a problem solving situation. It is also possible that the instrument used to measure language achievement in this study - a five-point scale based on teacher evaluation and creative writing - differs in important respects from Loban's measure and did not discriminate sufficiently between those with more or less power over language. What is probably needed is a re-examination of the manner in which language skills are assessed in the classroom. Halliday's (1969) concept of verbal skills based on an awareness of the various functions which language serves might be a useful tool for the purpose of evaluating language growth in children. It is possible that, as he suggests, pupils need to develop competence

in the particular language skills which are related to flexible strategies of problem solving.

The investigation showed that there were no significant differences between girls and boys in problem solving ability as measured by the Strategies of Problem Solving Test. This finding is consistent with those of other studies such as the one by Post (1967).

An interesting finding in the study was the extent to which successful problem-solvers used trial and error, or successive approximations, to solve a problem for which they had not previously learned an algorithm or standard method. Poor problem-solvers would often persist in using an inappropriate equation even when they were aware that it was not leading to a sensible or verifiable solution. The fact that use of an equation rarely led to a correct solution does not of course mean that equations are not a legitimate approach to arithmetic problem solving. There is no way of knowing to what extent previous experience with use of equations to solve problems may have contributed to a subject's successful use of trial-and-error in the present study.

One explanation for the failure of deduction to lead to a correct solution may be that the children in the study had not yet reached the stage of formal operations which, according to Piaget, is necessary before children can engage in logical reasoning and deduction. A further explanation for the success of trial-and-error has been suggested by Kilpatrick (1967). Trial-and-error may well be the best procedure to use when confronted with a higher-order problem for which no algorithmic solution is known.

The negative correlation of age with every other variable in

the study (Appendix D), including scores on the Strategies of Problem Solving Test, was perhaps the least important finding of the study; it was accounted for mostly by the fact that ten subjects (one-eighth of the sample) were on a five-year accelerated program in the school system. Therefore, the younger subjects, generally, were often the more intelligent.

In spite of the fact that the Strategies Test was designed to evaluate the process of problem solving while the Iowa test obviously measures only the product, significant correlations for the two tests indicate that there is a fair amount of overlap in the abilities measured by both tests. This is not surprising since both tests must certainly be dependent upon quantitative reasoning. The same reasons could also be given for the relationships between the two tests and verbal and non-verbal intelligence.

III. IMPLICATIONS

Several implications for education can be drawn from the findings. An inquiry approach to the teaching of arithmetic may be one way of helping children to develop flexible strategies of problem solving. Such strategies, however, can also be acquired in classrooms where a more traditional approach is combined with the modern activity methodologies. School systems might be well advised to study further the effects of the discovery or inquiry approach before urging teachers of elementary mathematics to adopt it. Nonetheless, it has been shown that the enthusiasm of the teacher using a particular method may be more important that the method itself. If the inquiry approach serves to stimulate enthusiasm on the part of the teacher and of the pupils,

then it most certainly is a 'good' method to use.

The investigation has shown that students in grade six can engage in solving problems not ordinarily included in arithmetic textbooks; problems which require the student to conjecture and to hypothesize. Teachers would be wise to pose problems of a "higher-order" from time to time and to refrain from "giving the answer", thus encouraging students to settle conjectures and verify their solutions.

A further implication is that the components of the strategies, sensing, predicting and verifying, could probably be applied to many problems already found in the grade six curriculum. More time should be spent in having students discuss the relationships involved in a problem, as well as in having them explore alternative methods of solution, and in encouraging them to ask constantly, "Can this answer be validated?"

An awareness of the different functions which language serves, particularly the heuristic function, may be important for the development of problem solving ability. The use of "higher-order" problems in the language classroom might be one way of increasing this awareness on the part of children and of developing the ability to think tentatively when confronted with a problem; an ability which this study has shown to be closely related to successful problem solving.

IV. SUGGESTIONS FOR FURTHER RESEARCH

This study used only one grade and investigated the effect of using an inquiry approach to the teaching of mathematics for one year only. Further studies should be made using more than one grade and/or examining the effect of using this approach over several years.

A second recommendation is that the effect of using trial-and-error as a method of solving arithmetic problems should be investigated, possibly by using a control group who were taught only to use deduction or algorithmic solutions.

The relationship of tentativeness to both problem solving and to the ability to use language effectively needs further exploration. Such studies should, however, attempt to distinguish between the different kinds of language and the functions which they serve.

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APPENDIX A

SELECTION OF THE POPULATION

MATHEMATICS SURVEY

The following questionnaire is part of a research project being planned by a graduate student in the Department of Elementary Education, University of Alberta. The survey will attempt to ascertain to what extent activity materials or a laboratory or non-textual nature are being used in the teaching of grade six arithmetic. All replies will be held in confidence and no results of the survey will be published without the express permission of the Edmonton Public School authorities. Thank you for your cooperation in the interest of research in education.

Name of teacher ________________________ School ________________________

Years of teacher training __________ Years of teaching experience _______

Arithmetic series used this year: _____ Seeing Through Arithmetic (Rev.)

_____ Elementary Mathematics (Addison-Wesley)

_____ Modern Mathematics (Holt-Rinehart)

_____ Other ________________________

Supplementary Activity or Laboratory Materials:

_____ Mathex

_____ Mathset

_____ Teacher-constructed

_____ Other

____

Approximate percentage of class time spent on activity or laboratory materials other than those contained in the textbook:

____ Seldom ____ About 25% ____ 50% ____ 75% or more

Additional Comments regarding arithmetic program:

OBSERVATION CHECKLIST

Teacher ______________________ School______________________

Use of textbook: Usually Sometimes Seldom Never

Type of activity: Class Small group Individual

Teacher-initiated Pupil-initiated

Exploratory Skill-practice Games

Kind of teaching: Expository Inductive

Generalization made by: Teacher Pupil

Number of questions asked: By pupils(tally)

By teacher(tally)

Pupil responses to teacher questions (tally)

Teacher responses to pupil questions (tally)

Teacher criticism of pupil's work:

Teacher praises pupil's work:

Concrete materials used: (list) ______________________

____

____

Other materials available ______________________

____

____

Teaching techniques used:______________________

____

____

APPENDIX B

COLLECTION OF DATA

LANGUAGE ASSIGNMENT

To be read to the class:

You will recall that Mrs. ________, who was a recent visitor to our classroom, is interested in finding out what kinds of things are interesting to boys and girls in grade six. She would like to know if you would help her by telling her about some of the things that interest you .

Since she can't possibly talk to all of you individually, she has asked me to have you write a paragraph for her on the topic:

SOMETHING I LIKE AND WHY I LIKE IT.

I will send her your paragraphs and I know she will read them all with a great deal of interest.

For the teacher:

1. Allow a discussion of some possible choices of activities or pets, etc. about which the students might write. (about 5 minutes)

2. Encourage students to choose a topic about which they will be able to write about 100 to 200 words. One side of a sheet of foolscap will be sufficient.

Since the ideas expressed by the children are more important than accurate spelling and/or neatness, do not allow children to re-copy their work.

Allow up to twenty minutes of actual writing time. Return the paragraphs in the envelope supplied for assessment by the writer prior to the testing date in your classroom. Thank you.

DIRECTIONS FOR ADMINISTERING IOWA TEST

1. Say a few words about the importance of the pupils doing their best work and that they should work quickly but carefully. "Your score will be the number of exercises for which you choose the right answer." Pass out scratch paper.

2. Pass out the answer sheets. Have the pupils fill in their name. Make sure each pupil has an HB pencil and knows how to fill in the spaces on the answer sheet. For practice, have them fill in the appropriate space for male or female. Point out that the answer which they choose for a question must be placed in the row that has the same number as the problem on the test.

3. Pass out the test booklets. Say, "DO NOT OPEN THE TEST BOOKLET UNTIL I SAY BEGIN." Read the directions aloud from the cover of the booklet and do the sample questions with the class.

4. "ARE THERE ANY QUESTIONS ABOUT THE TEST?"

5. "YOU WILL HAVE EXACTLY THIRTY MINUTES FOR THE TEST. YOU ARE TO WORK ALL OF THE EXERCISES ON THE TEST IN THIS WAY. DO NOT WASTE TIME REWORKING A PROBLEM IF YOUR ANSWER IS NOT LIKE ANY OF THE THREE SUGGESTED ANSWERS. INSTEAD, FILL IN THE SPACE MARKED "D" ON YOUR ANSWER SHEET AND GO ON TO THE NEXT EXERCISE. IF YOU COME TO A PROBLEM THAT YOU CANNOT WORK OUT, LEAVE IT AND GO ON TO THE NEXT ONE. IF YOU HAVE TIME, YOU MAY RETURN TO IT LATER."

6. " DO ALL YOUR WORK ON THE SCRATCH PAPER PROVIDED. MAKE NO MARKS AT ALL ON THE TEST BOOKLET."

7. "IS EVERYONE READY NOW?" (Pause a moment to make sure there are no more questions.)

8. "ALL RIGHT, BEGIN."

9. Circulate among pupils to make sure they are marking the answer sheet properly. Check also from time to time to see that pupils are not spending too much time on a difficult problem. If they are, suggest that they might be better to leave it and go on.

10. At the end of exactly 30 minutes say, " ALL RIGHT STOP. WOULD YOU ALL PUT YOUR PRENCILS DOWN AND CLOSE THE TEST BOOKLET."

11. Gather the test booklets immediately. Allow a moment for pupils to check that they have made all their marks heavy enough, and that any extra marks have been erased completely. Then, gather the answer sheets.

CRITERIA FOR SCORING WRITTEN COMPOSITIONS

Quality of Ideas, Organization and Presentation.

Value 5: The qualities suggested for 4 plus such added qualities as the happy turn of phrase, added polish, greater maturity of thought.

Value 4: Good selection of ideas.
Stays with the topic.
Paragraphing - paragraphs used when necessary.
Connectives used when necessary.
Sentences logically organized.
Reasonable variety in sentence structure to give a pleasing effect.
Pleasing and unpretentious style.
Agreeable as to appearance.

Value 3: The same qualities as in 4 above, but only average in effectiveness. Include papers otherwise well written but not on the topic assigned.

Value 2: Lacking in most of the qualities listed for 4, but intelligible.

Value 1: Generally lacking in the qualities for 4. Is incoherent, garbled, illogical, immature, obscure.

Mechanics and usage.

Value 5: No errors.

Value 4: 1 to 3 errors.

Value 3: 4 to 6 errors.

Value 2: 7 to 9 errors.

Value 1: More than 9 errors.

APPENDIX C

STRATEGIES OF PROBLEM SOLVING TEST

THE PROBLEMS

I. "the lost ball"

Just for practice let's begin with a game called "Twenty Questions." Here is a chart of a field (manilla tag marked off into 132 one-inch squares = 11 X 12). Let's say that a boy lost his baseball somewhere in this field. I know where it is, but you do not. You have twenty questions to see if you can find out where it is (Larson, 1966, p. 153).

II. "how many socks"

Jane was getting dressed for a school concert when a theunderstorm caused all the lights in the house to go out before she had picked out a pair of matching socks from her drawer. Her father is waiting in the car to drive her to the school. They have just enough time to get there before the concert starts. She knows that she has only yellow, white and blue socks in her drawer and that any of these colors will match the dress she is wearing. How many socks should Jane take with her so that she can be sure of having two socks the same color to put on in the car? (Explain that the socks are not arranged in pairs in the drawer. For the boys in the study Jane became "John" and the colors were blue, green and brown.)

III. "how much copper"

A nickel (five cents) is made of both nickel and copper. For every pound of nickel that goes into the mixture, there are three pounds of copper. How many pounds of copper would be needed to make 24 pounds of the nickel coins?

IV. "chess and checkers"

All of the students in a certain class know how to play either chess or checkers, but some know how to play both games. There are 14 students who know how to play chess; there are 16 students who can play checkers; but there are 10 students who can play both chess and checkers. Can you figure out how many students there are in the class?

V. "the vinegar problem"

A supermarket bought 275 gallons of vinegar in large barrels which had to be emptied right away and sent back to the factory. The manager of the supermarket found that he had only 168 empty gallon jars that he could fill with vinegar. He knew that these would not be enough to hold all the vinegar, so he decides to put the rest of the vinegar into pint containers. Could you figure out how many pint containers he will need?

VI. "father's age"

A man married at the age of 25. His wife died 15 years after their marriage, leaving a daughter who was only 11 years old. After 9 more years, the daughter married a man who was 4 years older than she was. When her father died, the daughter's husband was 45 years old. Is there any way that you could figure out how old her father was when he died?

VII. "gift of money"

Two brothers, Floyd and David, receive a gift of money each Christmas to be divided equally between the two boys. The total amount of money that their uncle sends for both of them is always equal to the product of their ages (explain the meaning of product). This past Christmas, in 1969, the uncle sent $36. The Christmas before last, in 1968, he sent $22 for the boys to divide between them. Could you figure out how old the boys were last Christmas; that is, what were their ages in 1969 when the uncle sent them $36?

QUESTIONS AND COMMENTS MADE BY EXPERIMENTER

Problem II

You may ask any question you like.
Can you tell me why you chose 4 (or 6, or 8)?
Is that the smallest number she can take and still be sure of getting two socks the same color?
Would she be sure or would she have to be lucky?
She has only white, yellow and blue socks in her drawer.

Problem III

You think they would need 8 lbs (or 72, or whatever) of copper to make 24 lbs. of the coins.
What do you think? Do you think the 24 lbs. would be both?
Could you explain how you got your answer?
I see.
Are you sure you've reached the right answer?
How do you know?
(Repeat the subject's statements at any time without expressing approval or disapproval.)

Problem IV

Why do you think there are 30 (or 20 or 40) students in the class?
Do you think the 10 would be included? (in answer to a question about the inclusion of those who play both)
Are you sure that is the number of students in the class?
Why are you sure (or not sure)?

Problem V

Why do you think you should multiply (or divide) 96 by 4, etc.?
Is there any way of figuring out how many pint containers he will need?
How did you get that?
(Repeat any statement made by the subject, or say I see in a noncommittal tone.)
Yes there are 8 pints in a gallon (or 4 quarts, etc.).
Are you sure you've reached the right answer?
Why are you sure (or not sure)?

Problem VI

Why did you add 25 and 15 (or 40 and 9, etc.)?
Would you like to read it again?
Do you think there is any way of figuring out how old the father was when he died?
How did you get that 21? Why would you add 21 to the 49, etc.?
Is there any way you could be sure that is the right answer?

Problem VII

Is there any way of figuring out how old they might have been when they got the $36 or the $22?

Are there any other possibilities?

Is there any way of knowing for sure that they really were 9 and 4 (or 6 and 6, or 3 and 12, etc.)?

If they were both 18, how much money would the uncle have sent?

Remember the uncle multiplies their ages to decide how much money he should send?

Are you sure you've reached the right answer?

Why are you sure? (or not sure)

Do you think there is any way of knowing for sure just how old they were?

CRITERIA FOR SCORING PROBLEM I "THE LOST BALL"

Schematic Strategy	Score
(a) highly efficient grouping to eliminate alternatives	6
(b) systematic elimination of alternatives by rows or sub-groups	5

Example: (Score of 6)

Subject's questions or comments	Experimenter's response
Let's say that was the line right down the middle; is it on this side?	No.
Then is it on this side?	Yes.
Is it in this half of that side?	No.
Is it in this half of that side?	No.
Then is it in this half (referring to 1/16th of the total area)?	No.
Is it in that square?	No.
That one?	That's right.

Thematic Strategy:

(a) organized cumulative imaginative questions dealing with a theme	4
(b) tendency toward irrelevant questions dealing with a theme.	3

Example: (Score of 3)

Did it land near the center?	What do you mean?
Well, over in here. (pointing)	No.
Did he kick it foul or something	No.
Is there a park around here?	Outside the field.
Were there any trees in the field?	No.
Was he eating his lunch when he lost it?	No.
If he went out to play, do you know where he went out?	The gate is over here.
Could he have laid it right here as he went out?	No, he didn't
Maybe he laid it along the path?	Where do you think the path is?
Down here? (Pointing)	Yes, that's it.
Did he lay it in this square?	No.
But he was laying it down when he went out?	Yes.
Is it in this square?	Yes, that's it.

Unsystematic Strategy:

(a) semi-random guessing - some grouping of squares or questions dealing with a row;	2
(b) random guessing - questions dealing with specific squares.	1

CRITERIA FOR SCORING STRATEGIES TEST

PROBLEM II "HOW MANY SOCKS"

Sensing:

		Score
a.	Subject shows by questions or comments that the problem is firmly grasped or structured to permit search for a solution: e.g. "He has several pair of each color, but he only has three colors. He wants to know how many socks to take so he'd be sure of getting a pair."	2
b.	Problem is only partially grasped; subject searches for solution without clearly understanding what is given and what is asked for: e.g. "He could feel the socks and take two that feel the same."	1
c.	Subject refuses to accept the situation as a problem he could solve; e.g. "There's no way of knowing he has two the same unless he takes them all."	0

Predicting:

a.	Subject considers various alternatives or possibilities; a valid hypothesis is made: e.g."if she takes 3 socks, they might be one of each color"; a systematic search is made which leads to a valid answer.	2
b.	Subject gives correct answer and then changes his mind; semi-random guessing; makes an invalid hypothesis: e.g. "if she took 6 they could all be different."	1
c.	Student indulges in random guessing; tendency towards irrelevant statements.	0

Validating:

a.	Gives a good reason for a correct answer: e.g. "if she takes 4 socks, they couldn't all be different because she only has 3 colors, so 4 is enough."	2
b.	Subject gives a valid reason for not accepting an incorrect answer: e.g. "She wouldn't need to take 8 because there are only 3 different colors, but I'm not sure how many she should take." Subject says that 4 socks would be enough but isn't quite sure why.	1
c.	No attempt to verify an answer; says there is just no way of knowing.	0

PROBLEM III "HOW MUCH COPPER"

Sensing:

Score

a. Student shows that problem situation is firmly grasped and structured to permit search for a solution: e.g. "The 24 lbs. of coins is made up of nickel and copper both; the recipe says 1 lb. of nickel to 3 lbs. of copper" or "I have to find out how much of this 24 lbs is copper." 2

b. Situation is only partially grasped; subject doesn't recognize that the 24 lbs. is made up of nickel and copper both, but knows that there has to be three times as much copper as there is nickel. 1

c. Subject randomly manipulates numbers without showing that he understands that the amount of copper must be three times that of the nickel. 0

Predicting:

a. Arrives at the correct answer, 18 lbs. of copper either by use of rate equation, deduction or some kind of approximation. 2

b. Suggests only inappropriate solutions but has shown that the problem is understood. 1

c. Refuses to predict a solution; says it can't be done. 0

Validating

a. Gives a valid reason for a correct solution: e.g. " 18 is 3 times 6 and 18 and 6 together come out to 24 lbs. so it has to be right." 2

b. Recognizes that 8 lbs. or 72 lbs. can't be correct because of a valid reason; but fails to come up with the correct solution. 1

c. Makes no attempt to validate a solution; says it has to be right because 24 times 3 is 72 or 24 divided by 3 is 8; relies on correctness of computation to verify an answer. 0

PROBLEM IV "CHESS AND CHECKERS"

Sensing:

		Score
a.	Recognizes that the 10 who play both chess and checkers are included in the 14 chess players and the 16 checker players.	2
b.	Has only a partial grasp of the situation; doesn't recognize that the 10 are included both groups.	1
c.	Doesn't recognize the problem as being one of inclusion.	0

Predicting:

a.	Reaches the correct solution by excluding the 10 from one of the other two groups, or by subtracting.	2
b.	Adds 14 and 16, but fails to subtract the 10.	1
c.	Just adds the three groups without knowing why.	0

Validating:

a.	Gives a valid reason for excluding or subtracting the 10.	2
b.	Can't give a reason for subtracting the ten; gives a reason for not including the 10 but is unsure and changes back to an incorrect response.	1
c.	Can't give any reason for adding or subtracting; has not come to grips with the problem.	0

PROBLEM V "THE VINEGAR PROBLEM"

Sensing:

a.	Recognizes the problem as requiring the changing of 96 quarts into gallons before subtracting from the gallons of vinegar and also that the remainder will be in gallons and must be changed into pints.	2
b.	Partially grasps the problem situation but fails to see the necessity of changing the units.	1
c.	Doesn't accept it as a problem; says he can't do it.	0

Predicting:

Score

a. Carries out all the necessary calculations to arrive at the correct solution. 2

b. Only does part of the required calculations or makes an error in computation or a small error in methodology. 1

c. Manipulates numbers in a random fashion without coming to grips with the problem. 0

Validating:

a. Gives a good reason for believing that 664 pints is the correct answer by explaining the necessity of changing the quarts to gallons and the gallons to pints. 2

b. Gives a poor reason for a correct solution: e.g. the correctness of the computation; or recognizes that an answer is incorrect but is unable to find another solution. 1

c. If the subject has engaged in random calculations showing no grasp of the problem, assign 0 for this section also. 0

PROBLEM VI "FATHER'S AGE"

Sensing:

a. Grasps the significance of the relationship between the son-in-law's age and the age of the father. 2

b. Only partially grasps the significance of the son-in-law's age as it relates to the age of the father. 1

c. No grasp of the problem; what is given and what is required. 0

Predicting:

a. Arrives at the correct answer, 70 years, by appropriate calculations. 2

b. Omits to do part of the calculation or makes a computational error. 1

c. Randomly adds numbers without understanding the problem. 0

Validating:

a. Gives a good reason for adding the 21 years to 49: e.g. 45 minus 24 is the number of years they were married; the father lived all that time too, so he must have been 21 years older too. 2

b. Partially validates the solution: e.g. gives reasons for bringing the father's age to 49 but is unable to proceed from there; or finds the right age but is unable to tell why the 21 years should be added. 1

c. Gives no reasons for adding numbers in a random fashion. 0

PROBLEM VII "THE GIFT"

Sensing:

a. Grasps the significance of the two amounts of money and understands what is asked for; also realizes that there is a relationship between the amount of money that is sent for two consecutive years: e.g. "I have to find two numbers that go into 36 evenly and when you subtract one from each of them and then multiply you'll get 22." 2

b. Partially grasps the problem; realizes the significance of finding two factors of 36 and 22 but doesn't see the relationship of the amounts in terms of two consecutive years. (No grasp of the problem at all, score 0) 1

Predicting:

a. Searches for and finds the correct combination of ages which will accommodate the one-year difference in their ages. 2

b. Suggests several combinations of ages but is unable to reach the correct solution. 1

c. Says that there is just no way of figuring the boys' ages when all you know is the money they received. 0

Validating:

a. Gives a good reason for believing that 3 and 12 are the right answers: e.g. 3 times 12 gives you $36 for 1969 and then if you take one year off each age and multiply you get 22." 2

b. Gives a valid reason for believing that suggested combinations are not the right ones, but is unable to find the correct solution: e.g. " 9 and 4 work all right for the $36, but not for the year before when they got $22, so it can't be right. 1

c. Is unable to find any way of checking to see if a combination that works for 1969, such as 9 and 4, are actually the ages that the boys were. 0

APPENDIX D

ADDITIONAL TABLES AND OTHER DATA

TABLE XXI

MEANS AND STANDARD DEVIATIONS FOR THE IOWA TEST: SUBGROUPS

(N=10)

Inquiry			Non-Inquiry		
Class	Mean	Standard Deviation	Class Code	Mean	Standard Deviation
Ia	14.0	5.2	Na	16.4	3.8
Ib	16.7	3.5	Nb	20.4	5.3
Ic	16.4	5.8	Nc	15.7	5.2
Id	16.9	5.5	Nd	20.0	4.2
Total	16.0	5.0	Total	18.1	5.0

One-way Analysis of Variance between total Inquiry and total Non-Inquiry Groups: F-Ratio 3.60 (d.f.1/78) Probability .06

Scheffé Multiple Comparison of Means for Subgroups: differences not significant at the .05 level.

TABLE XXII

INTERCORRELATIONS AMONG MAJOR VARIABLES

Variable	1	2	3	4	5	6
1. Age	1.000	-0.547**	-.0368**	-0.140	-0.262*	-0.267*
2. Verbal I. Q.	-0.547**	1.000	0.531**	0.526**	0.437**	0.352**
3. Non-Verbal I.Q.	-0.368**	0.531**	1.000	0.303**	0.508**	0.483**
4. Language	-0.140	0.526**	0.303**	1.000	0.469**	0.194
5. Iowa Test	-0.262*	0.437**	0.508**	0.469**	1.000	0.399**
6. Strategies Test	-.267*	0.352**	0.483**	0.194	0.399**	1.000

* Significant at .05 level

** Significant at .01 level

TABLE XXIII

DATA PERTAINING TO SUBJECTS FROM INQUIRY CLASSROOMS

Student Number	Sex	Age in Months	Verbal I.Q.	Non-Verbal I.Q.	Strategies Test Problem: II	III	IV	V	VI	VII	Total	Lang. Rating	Iowa Score
101	F	130	135	119	6	6	3	6	6	6	33	4	19
102	M	142	95	94	6	1	1	3	1	6	18	2	9
103	F	144	103	96	4	2	2	2	3	4	17	2	10
104	M	130	120	108	6	2	2	6	4	2	22	4	13
105	F	136	86	95	6	1	2	6	6	4	25	2	15
106	M	128	119	125	6	6	4	5	5	6	32	3	16
107	F	137	123	103	4	1	2	6	6	1	20	4	20
108	F	138	87	104	6	2	3	0	6	0	17	2	9
109	M	141	98	101	4	2	2	2	1	6	17	1	7
110	M	134	122	128	5	6	2	5	6	6	30	4	22
201	M	128	128	130	6	6	2	6	6	6	32	3	22
202	F	137	123	128	5	4	6	6	6	6	33	3	18
203	M	138	106	131	6	2	3	1	6	2	20	2	17
204	F	145	122	112	2	6	1	1	6	2	18	5	17
205	M	145	98	112	6	6	3	6	6	6	33	3	20
206	F	146	101	105	2	3	2	1	4	6	18	4	18
207	F	147	94	98	3	2	1	0	3	3	12	2	10
208	F	140	98	97	2	2	5	0	0	3	12	3	17
209	M	137	103	96	2	2	2	1	1	6	14	1	12
210	F	146	114	101	6	1	2	1	6	4	20	5	16

TABLE XXIII cont'd.

301	M	148	78	102	1	1	2	0	2	4	10	2	15
302	F	138	104	91	1	2	1	0	0	0	4	3	13
303	F	157	75	77	2	2	1	0	0	1	6	1	8
304	M	142	94	115	1	1	1	1	5	6	15	1	9
305	M	148	117	130	1	6	1	1	6	6	21	3	19
306	M	150	85	122	5	2	2	6	4	6	25	2	19
307	F	141	129	122	6	2	1	4	5	6	24	5	28
308	F	133	143	117	1	1	1	5	6	1	15	4	20
309	F	143	100	107	1	3	1	6	4	2	17	3	15
310	M	140	112	117	6	2	2	6	6	3	25	2	18
401	M	149	121	120	6	6	3	6	6	6	33	3	28
402	M	146	91	93	1	2	2	3	0	1	9	3	12
403	M	139	117	107	2	2	1	4	6	1	16	2	11
404	F	130	125	143	6	6	2	6	6	6	32	3	21
405	F	133	127	101	4	2	2	2	5	4	19	4	18
406	M	138	123	113	1	1	2	6	6	6	22	3	20
407	M	144	116	113	1	2	3	6	3	1	16	4	17
408	M	139	112	112	3	6	3	6	1	4	23	2	17
409	F.	147	85	82	2	2	6	0	0	0	10	2	9
410	F	151	88	99	3	2	2	6	6	1	20	1	16

TABLE XXIV

DATA PERTAINING TO SUBJECTS FROM NON-INQUIRY CLASSROOMS

Student Number	Sex	Age in Months	Verbal I.Q.	Non-Verbal I.Q.	Strategies Test Problem: II	III	IV	V	VI	VII	Total	Lang. Rating	Iowa Score
501	F	139	116	134	6	6	2	6	6	2	28	4	19
502	F	141	114	111	6	6	2	2	6	6	28	5	16
503	M	155	104	95	2	1	1	1	4	0	9	3	10
504	F	143	104	112	5	1	2	2	4	3	17	3	20
505	M	144	105	109	0	1	1	3	6	0	11	4	14
506	M	140	108	109	1	6	4	5	3	6	25	2	18
507	F	126	124	127	6	2	1	0	3	0	12	5	22
508	M	142	108	107	1	1	1	5	0	1	9	4	14
509	M	146	94	106	6	2	6	3	5	0	22	2	12
510	F	142	114	113	1	3	2	1	0	0	7	5	19
601	F	146	91	118	1	3	5	6	6	6	27	3	24
602	M	142	103	118	6	6	2	3	6	6	29	4	22
603	F	147	90	108	1	1	1	1	4	4	12	4	28
604	F	139	131	118	6	4	6	6	3	1	26	3	24
605	M	139	113	120	2	3	1	1	6	0	13	2	11
606	M	138	101	104	6	1	4	6	0	2	19	1	22
607	F	146	111	122	5	2	2	5	1	0	15	5	22
608	F	137	104	107	1	6	6	1	1	3	18	3	15
609	M	148	118	105	1	6	1	4	6	6	24	5	22

TABLE XXIV cont'd.

610	F	145	99	104	5	3	2	5	3	1	19	4	14
701	M	144	116	108	1	2	2	5	1	6	17	4	19
702	M	153	108	111	0	1	4	3	1	0	9	1	7
703	F	143	110	109	3	0	2	0	1	1	7	1	13
704	F	139	106	100	2	0	1	1	0	0	4	3	10
705	F	129	126	98	2	4	3	6	1	1	17	4	21
706	M	143	103	108	2	1	2	6	6	6	23	3	19
707	M	153	100	84	1	1	1	0	1	0	4	3	11
708	F	140	105	112	6	2	1	1	1	0	11	2	18
709	M	143	94	114	1	1	1	1	1	0	5	2	23
710	F	141	97	117	1	1	1	0	1	0	4	3	16
801	M	148	124	117	6	6	2	6	6	6	32	5	19
802	M	136	93	86	6	1	6	6	4	0	23	1	17
803	F	144	92	100	0	2	1	0	0	2	5	2	13
804	F	136	128	102	5	1	2	4	4	4	20	3	21
805	F	140	112	111	1	1	2	2	5	0	11	4	16
806	F	142	95	118	1	2	1	3	6	3	16	3	19
807	M	139	98	98	1	6	2	1	3	2	15	2	21
808	M	130	128	120	1	0	1	2	1	1	6	3	28
809	F	139	140	119	1	2	4	3	6	2	18	4	23
810	M	147	91	97	0	1	6	5	1	6	19	4	23

APPENDIX E

TENTATIVE AND NON-TENTATIVE PROTOCOLS

EXAMPLES OF TENTATIVE PROTOCOLS

Problem III (how much copper)

(a)

Subject's Comments or Questions	Experimenter's Comments
I think I know how many. Six.	You think 6 lbs. of copper.
Oh, no! Six of those 3 lb. chunks, so that would be 18 of copper.	Why do say 18?
Well, 6 times 3 is 18, and 6 times 1 is 6, and 18 and 6 makes 24 lbs. altogether.	Where did the 6 come from?
Well I tried different other numbers.	What other numbers did you try?
I tried 4 and it was too short 'cause 12 and 4 is only 16, so I went to 6 and it worked.	

(b)

Subject's Comments or Questions	Experimenter's Comments
Three times 24 is 72. I guess it would be 72 lbs. of copper.	You think 72 lbs of c copper.
Yes, but, wait, where would the nickel be? That can't be right because you have to put the nickel in too.	I see.
Every pound, one pound for every... (pause)	Is there enough information given?
Yes. (long pause) I keep getting 8 but I don't know where it comes from.	You don't know how you got 8?
Well maybe it could be used in the copper but how did I get that when I need the nickel?	You need the nickel?
Yes, I got 8 because I divided 24 by 3, but I have to see how much the nickel is.	Would 8 lbs. of copper be needed to make 24 lbs. of the coins?
It could be, but somehow nickel doesn't fit in there by three it needs, and yet,72 pounds is too much. Let's see... there's one lb. of nickel and 3 lbs. of copper and you keep doing that until you get to 24. (She writes: 2 6 / 3 9 / 4 12)	
That's it I think.	Is that 4 of nickel and 12 of copper?
Yes, but that still isn't enough...	Why not?
Because 4 and 12 is only 16. (At this point the subject gave up and said the answer must be 72lbs. because 3 times 24 made 72.)	

Problem VII (gift of money)

(a)

Could they both be 18?	What do you think? Could they?
No, I don't think so, because 18 times 18 is a lot more than 36.	Is there any other way you could figure out their ages?
Well, it could be 6 times 6.	Why do you say that?
Because that's just one age they could have been.	Is there any way of knowing for sure?
Well, one could have been 4 and the other one 9.	I see.
But then it would have to have been $25 they got in 1968.	Why?
Because they would have been 5 years old then.	What does that mean?
I suppose I could do the 22 first and then the 36 next. And you have to times the two ages.... I suppose they could have been 11, no... that's adding. Could one be older?	I imagine he could.
The closest thing to that (indicating 4 and 9) would be 3 and 7...I don't think the uncle would send an extra dollar.. (He re-reads the problem.)	
Four and five, three and seven.	What are you thinking about?
I'm looking for two numbers that multiply up to 22.. I don't think there are any.	Why do you want two numbers that multiply up to 22?
Well, then I could add one to each of them and then see if it came to 36.. and if it did then you would know that was their ages.	I see.
Oh! one could be 1 and the other 22... but that would mean that next Christmas it would be 2 times 23 and that's not right.	Is there enough information given?
Yes, I think so. But how did he get that?	Get what?
The twenty-two dollars. (pause) It could be 2 years old and then 11 for the other boy.	So how old were they?
Yes, it's two times 11 because then they'd be 3 and 12 the next year and 3 times 12 is 36...	Do you think 3 and 12 were their ages in 1969?
Yes, because it works for both years.	I see.

(b)

(He starts to multiply 3 times 13.)	What are you multiplying 3 and 13 for?
Because they could have been 2 and 11 in 1968. Oh, 3 and 12 it is.. that's it. They were 2 and 11 the first year and	

3 and 12 the next year. | Do you think 3 and 12 are the only possibilities?

Yes, because... like... I tried some other numbers in my head and they they don't give you 22, but 2 and 11 does... You see, I added one year to each of them and that gave me 3 and 12 and it worked out to $36. | I see.

(Note that although he reached the answer almost immediately, he had tried other numbers and was prepared to reject an answer that could not be verified -"didn't work out"- and therefore this subject was classed as tentative.)

(c)

I figured that 4 nines would work out to $36 but then if they have to be the year before... well, they'd be 3 and 8 and it doesn't come out to $22... | It doesn't work out.

Let's see, another possibility for the 36 is 3 and 12. | Why do you think 3 and 12?

Because 3 times 12 is 36 and then the year before they'd be 11 and 2. Yes, that works out for the $22. So they must have been 3 and 12 last Christmas. | I see.

EXAMPLES OF NON-TENTATIVE PROTOCOLS

<u>Problem</u> <u>III</u> (how much copper)

(a)

How much copper they would need?

Yes, to make 24 lbs. of the coins.

Eight pounds.

How did you get 8?

By dividing it.

Why do you think you should divide 24 by 3?

It seems like a good idea.

Is there any way of checking to be sure 8 is right?

Well, 8 times 3, and that gives 24, so it's got to be right.

Do you think there might be any other way of ' figuring it out?

Nope.

(b)

Is the 24 lbs. straight nickel?

The 24 lbs. is five cent coins. Are they straight nickel?

No, they'd be nickel and copper mixed. (He does some computation on paper.) They'd need 8 lbs. of copper.

You think they would need 8 lbs. of copper to make 24 lbs. of nickel coins.

Yes, you see I figured it out with this ratio. Like the lbs. of nickel are 1 and the lbs. of copper are 3... and that equals 24 lbs. of nickel and 8 lbs. of copper. (He has written 1/3= 24/8.)

I see. Are you quite sure then that they'd need 8 lbs. of copper.

Yes, cause I used the ratio and that's the <u>only</u> <u>way</u> <u>we've</u> <u>been</u> <u>taught</u> to do it.

<u>Problem</u> <u>VII</u> (gift of money)

(a)

I think I should be able to do this one. Let's see, oh yes, now, 36, you times something into that... I just have to figure out what. Six times six...that would work. Yes, they must be 6 years old.

You think they were both 6 years old.

Yes, and in 1968... they'd have been 5 years then... Hmmm... 5 times 5 doesn't equal 22.	What does that mean?
That I can't get this question, because it looks okay and I think it's working out and then I look at this one and it's not right.	Can you think of any other possibilities for their ages?
No, I can't do it. I think this is a funny problem.	You can't think of any other possibilities?
Nope.	

(b)

In 1969, they would each be 6.	Why do you think they were 6?
Because 6 times 6 is 36 and they got 36 dollars.	How about in 1968?
I can't figure that out.	I see. Are there any other possibilities for 1969?
No, just 6 and 6.	Is there any way of knowing for sure that they were six in 1969?
No, there's no way.	

(c)

Could they be twins?	What do you think? How old would they be then?
Well, 6 and 6... and then that would be 5 and 5. It doesn't fit.	Why not?
Because 5 times 5 is 25.	Is there any way of working this problem?
No there isn't, cause the 6 and 6 doesn't work for the 22, so I wouldn't know what to try next.	Is there enough information given?
No, I think they don't give you enough facts.	I see.